City-system dynamics in world history studied by change in city-size distributions[1]

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Abstract. Oscillatory patterns of expansion/contraction have long characterized the dynamics of demographic, economic, and political processes of human societies, including those of exchange economies and globalization. Major perturbations in city-size distributions are shown to exist for major regions in Eurasia in the last millennium and to exhibit some of the characteristics of cyclical oscillations on the scale of 100s of years as well as longer fluctuations, from 400 up to 800 years, between periods of major collapse, often punctuated by lesser collapse. Variations in timing, irregularities in amplitudes, and ups and downs in our measures appear to correlate with some of the peaks and troughs in urban population growth and show long-cycle correlations with J.S. Lee's (1931) sociopolitical instability (SPI) data on the durations of internecine wars for China.

We focus here on central civilization within the world cities database, including China and Europe, and the Mid-Asian region between. These data are likely to reflect changes in the macro regions connected by trade networks, where we would expect synchronization. Our interpretation of city-size distribution oscillations is that they follow, with generational time lags, rises and falls in the expansion/contraction of multi-connected trade network macro zones, with Zipfian city-size hierarchies tending to rise with trade network expansions and fall with contractions. City system rise and fall also tend to couple with oscillations of population relative to resources interacting with SPI in total cycles that average about 220 years. Time-lagged synchronies in the dating of phases for city distributions in different regions that are connected by multiple routes of trade, as noted tentatively by Chase-Dunn and Manning (2002:21), at least in the rising and more Zipfian phase, support the existence of city-system rise and fall cycling. We find evidence that rise and fall in Silk Road connectivities between China and Europe had time lagged effects on the growth of power law tails in European urban hierarchies; that changes in Mid-Asia city distributions led weakly those in China while those in China led strongly those in Europe, at different time lags.

Maximum likelihood of two different measures of city size distributions proved to be of central importance to this paper, as they provide unbiased estimates of statistical parameters and improve confidence limits significantly, the more so for those smaller sample sizes in city data available in many historical periods. Analyses of these estimates supports six major sets of hypotheses about urban system evolution and fail to contradict two more speculative hypotheses about evolutionary learning in global systems.

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0. Introduction

Globalization, world-system, and historical dynamic theory offer complementary perspectives for the study of city systems as the politicoeconomic engine of interstate networks. Globalization theory, applied to Eurasia in the last millennium (e.g., Modelski and Thompson 1996), focuses on centers of economic innovation, political power, and successive rise and fall in dominance. Units of larger scale, such as polities, are shown to operate at successively longer time scales in rise and fall than the economic innovation centers within. World-system theory (e.g., Chase-Dunn and Hall 1997) differs in focusing at the peripheries of states and empires, that being on the marcher or boundary polities, resisting the encroachment of expanding empires. Defeating the spread of empire through superior cohesion, decentralized organization, and superior combative skill or technology, amalgamated marcher states often defeating formally organized polities on a much larger scale. World-systems theory is often limited to the more prominent types of relations, such as trade in bulk goods and interstate conflict that form distinct macroregional networks. The structural demographic approach to the political economics of agrarian empires (e.g., Turchin 2003b, 2006) is capable of yielding a more dynamical historical account of how central polities rise and fall as their internal cohesion disintegrates with population growth into factional conflict and of how once dominant polities and economies contend with marcher states that coalesce into formidable opponents on their frontiers.

Several of the problems in extending these kinds of complementary approaches to globalization, world-systems, and historical dynamics relate to how networks – social, political, and economic – fit into the processes of change and dynamical patterns that are observed historically. One major problem in network research involves how network fluctuations of long distance trade influence inter- and intra-regional dynamics. For example, the Silk Road is important in the connections through the marcher states and later empires of Mongol Central Asia between China and the Middle East, facilitating the diffusion of economic inventions, such as paper money, institutions of credit, and vast new knowledge, weaponry, and technologies, from East to West and was crucial in the rise of the European city system. Spufford (2002), for example, shows the importance of transmissions of innovations from China from a European perspective, while Temple (1987) summarizes the work of Needham (1954-2004) to show the debt of the West to China.

Among many other open network related problems in historical research is the coupling of regional and long-distance trade networks, with conflicts and wars to the rise and fall of cities and city systems and the historical dynamics of globalization and world-system interactions in Eurasia. For example, changes in one region, such as China affect changes in others, such as Europe. During the last millennium, with a concern for valid comparative measurement of large scale phenomena, Tertius Chandler (1987) and other students of historical city sizes (Pasciuti 2006, Modelski 2003, Bairoch 1958, Braudel 1992, and others) have made it possible to compare the shapes of city-size distribution curves.

Our approach here is to divide up Chandler's (1987) Eurasian largest city-sizes data into 3 large regions – China, Europe, and the Mid-Asian in between – and measure variations over time that depart from the Zipfian rank-size distribution. Zipfian rank-size is the tendency for cities ranked 1 to n in size to approximate a size of M/r, where r is a city's rank compared to the largest city and M is a maximum city size that best fits the entire distribution (this formulation allows the rank 1 largest city size S_1 to differ from its expected value under a Zipfian fitted to an extensive set of the larger cities). The Zipfian distribution has been taken to be a recurrent and possibly universal pattern for city sizes as well as many other complex system phenomena. We find that for Eurasia and regions within it that there are systematic significant deviations from the Zipfian in some historical periods that show the characteristics of a regional collapse of city systems from which there is eventual recovery (unlike cataclysmic collapse exemplified by the Mayan cities system).

Each of 3 Eurasian regions has different periods of rise and fall for city systems for, allowing us to test the hypothesis that the rise and fall measure for China anticipates with a time lag those for Europe. The period starts at 900 CE consistent with Modelski and Thompson (1996), Temple (1987) and Needham (1954-2004). Finally, for the region of China, we have sufficient time-series data to test the predictions from the historical dynamics model of Turchin (2005). This allows some limited results on whether some of the same processes are operative for the rise and fall of and historical dynamics of city systems, states, and empires.

Part 1 poses the problem of instabilities in city sizes and systems drawing on Chandler's data for 26 historical periods from 900 CE to 1970. Part 2 examines ways of measuring departure from Zipfian distributions of city sizes and introduces the data used for city sizes and possible correlates of city system change. Part 3 gives the results of the scaling of city sizes for different regions so as to measure city system changes. Part 4 examines the time-lagged interregional cross-correlations and summarizes the cross-region synchrony. Part 5 examines correlations and time-lags between our 3 Eurasian regions and for other variables related to known historical oscillations with adequate data for hypothesis tests. The variables tested include trade connectivity, internecine warfare within China and development of credit and currency systems that facilitate international exchange as well as innovative national markets. Part 6 concludes with a summary and implications of the findings.

1. City System Instabilities

Jen (2005:8-9) defines an equilibrium state to have *stability* if dynamical recoveries from small perturbations return to the original state. She defines *structural stability* as the ability to return from instability through other dynamics than the original (e.g., by varying external parameters) that are qualitatively similar to the original dynamic. While economic and political systems are not stable in the strict sense, they may have the resilience to return to structural stabilities if they pass through some sort of oscillatory or feedback fluctuations with differing, but qualitatively similar dynamics (for useful discussions for population dynamics in trophic interactions, see Turchin 2003a:78-159, and for more general endogenous feedback dynamics in human population dynamics see Turchin 2003b). However, major population growth trends interacting with dynamical oscillations or limit cycles may lead to *structural instability*: an inability to return to stability, including through other qualitatively similar dynamics. The two main factors that make for instability are competition and population growth. Economic competition, aided by power politics, tends to make for oscillations that may return to what might be called structural stability, making for economic and political limit cycles rather than conservative stationary. Populations of polities, empires, regions, and global world systems, also exhibit limit cycles if we average out trends of population growth, e.g., over the last several millennia. However, incessant competitive innovation for successful cities and city systems leads to population overgrow relative to resources and to subsequent system crashes. Historically, these instabilities lead to industrial revolutions that, rather than conserve materials and energies, and may push extravagant degradation of resources into dynamically irreversible crises such as global warming. Unless innovation turns toward conservation the problems created might not be solved in the next century or possibly not in next millennium. The issues here are ones of scale, expansions of scale (size of cities, size of polities and empires, size of economies), the dynamic interactions that operate at different scales, and how these couple spatially and temporally (as described, for example, in Modelski and Thompson 1996).

Our first questions are the stability of city systems as central economic actors and sites for multitudes of agents. If unstable, what kinds of models are appropriate for consistency with their dynamics? The thesis is not simply that individual cities grow and decline, but that entire regional (and global) city systems do so. Here, drawing on our earlier work (White, Kejžar, Tsallis and Rozenblat 2005), Michael Batty (2006:592) states our case for us. “It is now clear that the evident macro-stability in such distributions” as urban rank-size or Zipfian hierarchies at different times “mask a volatile and often turbulent micro-dynamics, in which objects can change their position or rank-order rapidly while their aggregate distribution appears quite stable….” Further, “Our results destroy any notion that rank-size scaling is universal… [they] show cities and civilizations rising and falling in size at many times and on many scales.” What Batty shows, using the same data as do we for historical cities (Chandler 1987), is legions of cities in the top echelons of city rank being swept away as they are replaced by competitors, largely from other regions.[5]

2. Data

2.1 City Size Data for Historical Eurasia

Chandler’s (1987) database on historical city sizes is complemented by overlapping UN population data from 1950 to the present (in the interest of brevity we do not present these results here). Chandler reconstructed urban populations from many data sources. These included areas within city walls times number per unit area, connected house-to-house suburbs lying outside the municipal area, data from city histories provided by city librarians, estimates from numbers of houses times numbers per house, and the cross-checking of different estimates (see Pasciuti and Chase-Dunn 2002). From 900 CE to 1970 his size estimates cover over 26 historical periods, usually spaced at 50 year intervals, always comprises a set of largest cities suitable for scaling in a single period. These data include 80 Chinese, 91 European, and in between a much larger number of Mid-Asian cities.

Figure 1 show numbers of cities in the dataset for in each period when they fall below 21. European cities in the top 75 world cities rise from 8 to 21 from 1100-1575, while those of China drop from 19 to 11 between 1200

[5] Noting from the shared database that the top echelon of cities in a single region may be swept away in a short period by interregional competition, Batty refers to our work on instabilities at the level of city systems.

and 1650 CE. Mid-Asia has more than 20 cities in the top 75 up to 1875. One concern is whether there are too few cities in some periods to differentiate characteristics of the tail of the size distribution (largest cities) from that part of the size distribution that reaches down to smaller cities.

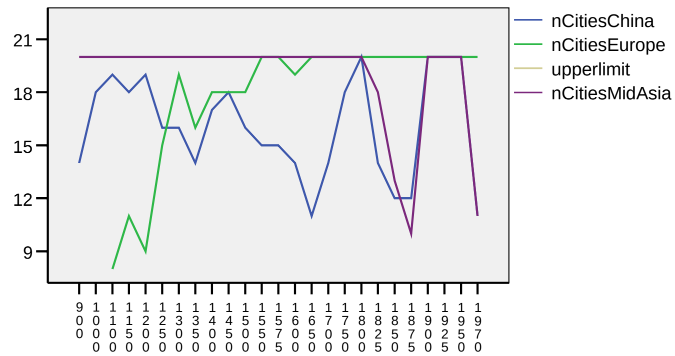

Figure 1: Number of Cities in Top 75 World Cities in each region (shown) when they fall below 21

2.2 Trade Routes (Eurasia). The total length of the Eurasian long-distance trade routes between 3500 BCE and 1500 CE at 50 year intervals have been calculated by Ciolek (2005) from trade-route maps drawn by Sherratt (2003). World-system pulsations in expansion and contraction of trade routes are shown by Turchin (2007). Turchin calculated a connectivity index for the Silk Routes between China and England. His data also show how instances of epidemics are concentrated near the high points of trade before periods of collapse.

2.3 Sociopolitical Instability: Internecine Wars (China only)
Turchin (2003b:164) transcribed J. S. Lee's (1931) coded 5-year interval data on internecine wars in China, ranging from regional uprisings to wide-spread rebellions and civil wars, from 221 BCE (unification by the first Ch'in Dynasty Emperor) to 1929 to create a ten-year sequential intensity index to 1710. D.R. White converted Turchin's codes into a 25-year index running from CE to 1700 and coded 1725-1925 directly from Lee (1931). Lee coded the period to the end of the Ming Dynasty from a remarkably systematic inventory of conflicts by Chih Shao-nan from the Tih Wang Nien Piao and checked for accuracy by Tung-Kien (Lee 1931:114). The Tabula Annua (Seikainenkan), cross-checked and supplemented by the Tai Ping Tien Kuo Chan Ssi proved a reliable record of Ch'ing Dynasty wars, with those following from Lee's memory. The systematic pattern discussed by Lee for his graphs is one of two 800-year periods (200 BCE-600CE, then to 1385 CE) in which many more of the intra-territorial China conflicts occur in the last 400-500 than in the early 400-300 years, and a partial repetition up to 1927 of that same pattern. The other evident pattern is for conflicts to become much more likely at the transitions between dynastic periods.

2.4 Total Population and Population Change Data (Eurasia)
The early data on Chinese population from 900 to 1300 are controversial. White had data from Chao and Hsieh (1988), provided by Turchin (2003b:164-165), he consulted Ho (1956, 1959), Steurmer (1980), Mi Hong (1992), Durand (1960), Heilig (1997, 1999, 2002), and the radical revision proposed by Heijdra (1995) and Mote (1999) that was critiqued by Marks (2002), and others. Given the uncertainty in absolute figures, we coded a binary variable for each 25-year period where 1 is given for a date at which there is a population peak before collapse, with 0 otherwise. The different total population estimates available to us for China over our full time frame agreed very closely as to where these population peaks occurred. In some cases two adjacent peaks were indicated.

Turchin (2006, 2007) provided population, carrying capacity, detrended population, and a misery index (inverse wages) for England that could be useful in comparisons with our European city data.

2.5 Monetary Liquidity (China only)
We coded for 900-1700 an index of monetization (liquidity) in China using Temple's (1986:117-119) discussions (drawing on Needham 1954-) on the development of credit, paper money, banking, and inflation (indexing lower liquidity) into a qualitative judgmental scale from 0-10.

3. The q/β Scaling and Hypotheses
3.1 Measuring Departures from Zipf's Law for City Size Distributions

We begin by examining the instability of city size distributions of macroregions in the Eurasian continent, over roughly the last millennium, when planetary globalization emerged, by visually inspecting changes of the shapes indices of Eurasian city size distributions. Figure 2 shows a semilog graph of the cumulative rank-size distribution for divisions of most of Eurasia (excluding Japan/Korea) into three regions: China (c900-c1970), Europe (e900-e1970) and the Mid-Asian (m900-m1970) remainder. The curve to which power-law distributions should correspond is shown by the top (ZipfCum) power-law curve. Cumulative population size is logged on the y axis and the x axis is city size rank. The Zipfian curve forms a straight line when rank is also logged, with a Pareto log-log slope of 2. As can be seen visually, there is some departure from perfect parallelism in the empirical curves: some lines are more curved or less curved for the top cities than the Zipfian, most lines are flatter that the Zipfian for the smaller cities and many of the curves bend at different city ranks.

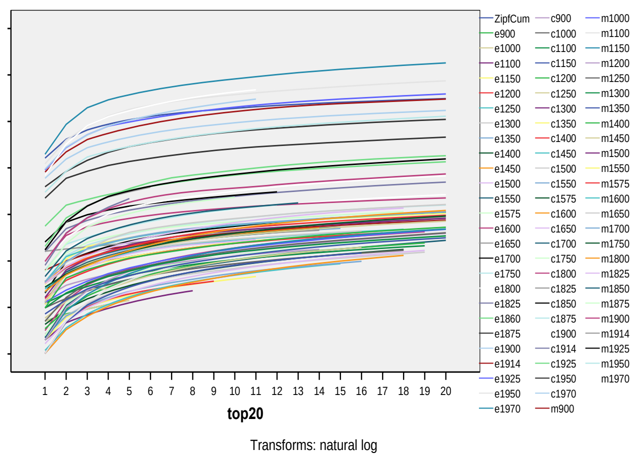

Figure 2: The Chandler Rank-Size City Data (semilog) for Eurasia

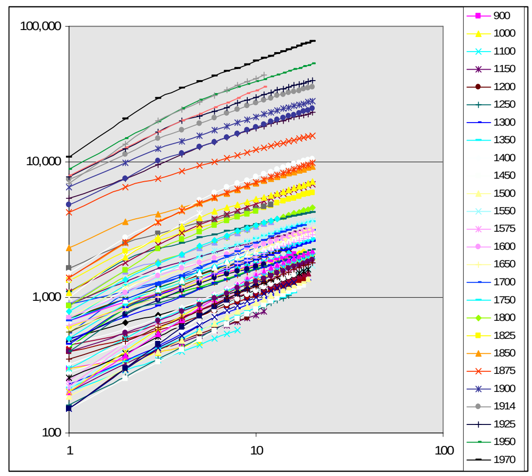

Figure 3: The Chandler Rank-Size City Data (log-log) for Eurasia

Figure 3 shows the same data (all 3 regions) in a log-log plot where power-law city distributions would all be straight lines and Zipfian distributions would all have the same slope. The lines are neither parallel nor of the same slope, nor do they have curvatures in the same places. Our measurements of the properties of these distributions will be aimed at the hypothesis that these variations provide indicators useful to showing how city system fluctuations fit into economic and historical dynamics.

We use two measures of how these curves vary. Each measurement model is based on a standard continuous function with parameter fitting based on recasting the data as a cumulative probability in its complementary form. $P(X \geq x)$ is the probability that an urbanite will reside in a city of at least size x. Each model is aimed at capturing the complete (Pareto II) shape of the empirical $P(X \geq x)$ or part of that shape (Pareto I for the tail) as a cumulative complementary distribution function (CCDF). It is important to use Maximum likelihood estimates (MLE) in fitting the parameters of distributional models, and especially so with small samples. MLE estimates in this case are unbiased, meaning that the expected values of the estimated parameters for *n* independent samples of the same data would converge to the true parameter values.

The first measure is the standard (type I) Pareto distribution with a single parameter β where the Zipfian is the special case of β =2:

$$P_\beta(X \geq x) \sim x^{-\beta} \qquad (1)$$

Fit to this distribution captures the extent to which the lines are straight in Figure 3, as is typical for the tails of city size distributions.

Fit to a second function, a type II generalized Pareto distribution with a cut-off σ (Arnold 1983), captures the extent to which a given log-log distribution is curved. This function allows recognition that Zipf's and power laws for city sizes almost always have one or more cut-offs of a lower size below which the power-law changes or ceases to apply.[6] This function fits to the urban distribution a shape parameter Θ, analogous to β, and a scale parameter σ.

$$P_{\Theta,\sigma}\,(X \geq x) \sim (1 + x/\sigma)^{-\Theta} \qquad (2)$$

The type II Pareto has an extra cut-off parameter σ, but with unbiased MLE of parameters there are many advantages of a parameter σ that specifies the crossover where the curve breaks for a power-law or Zipfian tail. The Pareto II function:

a) fits an exponential function or a collapsed tail in cases where a power-law or Zipfian tail is inapplicable.
b) with MLE, yields parameter estimates that are unbiased by sample size.[7]

Unbiased MLE allows 1-to-1 functional substitutions of model parameters into equivalent models (Brown, 1986). The *q*-exponential $Y_q(S \geq x) = Y0\ (1-(1-q)x/\kappa)^{1/(1-q)}$ and its shape (q) and scale (κ—kappa) parameters is a 1-to-1 functional transformation of the unnormalized version of equation (2) where q=1+1/Θ and $\kappa = \sigma/\Theta$: $Y_q(S \geq x) = Y0\ (1\text{-}x/\sigma\ )^{-\Theta}$. Y_q has been previously used (Malacarne, Mendes, and Lenzi 2001) for city-size distribution scaling. Bootstrap estimates of the standard error and confidence limits of the q,κ parameters derived from Θ,σ are provided by Shalizi's (2007) R program for MLE. There are many other advantages of Y_q and Pareto II that we do not exploit here.[8]

A continuity theorem often used with the Y_q model (Tsallis 1988) gives the relation in the Pareto II and Y_q functions, when $q>1$, between a Y_q tail that asymptotes to a power-law with slope 1/(q-1) and a Pareto I tail with slope β. The Y_q curvature, however, captures that of the smaller cities in the distribution. The theorem is important to the study of city sizes where smaller size cities, which often lacking of data, do not follow a power law. Moreover, the *q*-exponential is interpretable as one of the theories developed in "behavioral statistical physics" (Farmer 2007) and q=1 (the limiting case of an infinite Θ) has a special meaning as an entropic system "at rest," lacking complex interactions and power-law behaviors. For cities, $q \leq 1$ is an indicator of system collapse!

3.2 Hypotheses

Several linked hypotheses build on one another, each supposing the previous hypotheses to be supported, and each adding greater specificity in relation to the parameters of the two models, β for the Pareto I and q for the *q*-exponential (thus Θ) for Pareto II:

H1. The Zipfian (β=2 and q=1.5 for our CDF) is posited as the most likely historical norm for the tails (β) and bodies (q) of city size distributions.[9] Over long historical periods these should be expected as average values around which q and β fluctuate.

H2. Variations in q and β are *conservative* as population measures affected by births and normal mortality but may change quickly when influenced by migration, and sociopolitical instability (SPI), that is, internecine wars or outbreaks of violence.

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[6] For example, excluding two primate city outliers, the next largest 16 cities for 1998 in the U.S. (over 11 million) show a steep log-log slope, those ranking down to .5 million show a shallower slope, those to 1 million a much shallower slope, and then the power-law disappears altogether (Malacarne et al. 2001:2).

[7] MLE runs can be batched for multiple datasets. Standard errors are typically very small for city size data and the true values of the parameters are likely to be within these limits. A typical batch of instructions using R, for example, might be:

```
china.900 <- c(500,150,90,81,75,75,70,65,60,58,49,47,40,40)
china.900.tsal.fit <- tsal.fit(china.900,xmin=40) # Assigns the results of the fit to the object
china.900.tsal.fit # Displays the estimated parameters and information about the fit
(these estimates run in seconds, and those following run in minutes)
china.900.tsal.errors <- tsal.bootstrap.errors(china.900.tsal.fit,reps=100)
china.900.tsal.errors # Displays the bootstrapped error estimates.
```

[8] These include:

a) Y_q estimates an expected "largest city size" M consistent with the body of the size distribution. This requires simultaneous estimation of M and Y(0) to solve Y(0) $P_{\Theta,\sigma}\,(X \geq M)=M$.
b) The total urban population can be estimated from Y_q without having data on all smaller cities, although this feature is not utilized here.
c) Equation (1) and Y_q may be fitted *without* the largest city so as to derive an expected size for the largest city given our model.
d) This gives our model a ratio measure of the largest city size to its expected size from Y_q.
e) Y_q has a known derivative $Y_q'(x)=Y0/\kappa[1-(1-q)x/\kappa]^{(q/(1-q))}$ giving the slope of the curve $Y_q(x)$ for any city size x.
f) Solving for $Y_q(M)=M$ for the estimated largest city size M consistent with Y_q gives $Y_q'(M)$ as the slope of the Y_q at M and converges with $\beta=1/(q\text{-}1)$ for the Pareto power-law slope.

[9] Elsewhere we estimates of the total urban population asymptote, Y(0), from fitting the Pareto II and equivalent *q*-exponential distributions.

H3. Variations in q and β are thus likely to exhibit stability within historical periods of multiple generations, with Zipfian values on both measures correlated with periods of stability and normality, followed by instabilities that may occur suddenly.

H4. As such q and β are indicators of rise and fall of urban system size distributions in both the body and the tail of the distributions, which may vary with considerable independence.

Tails may a) collapse (shorten, q dropping toward or beyond 1), or grow into (b) longer (thinner, lower β slope) tails, (c) Zipfian (β=2), or (d) shorter power-law (thicker, higher β slope) tails.

4a) Collapse of tails should co-occur with SPI, severe economic/political crisis, or major wars.

4b) Longer tails should be enhanced by capitals of empires and by exceptional centers of international trade that serve as economic magnets for migration from impoverished rural areas.

4c) Zipfian tails should be enhanced by intraregional trade with positive urban-hierarchy feedbacks.

4d) Shorter tails should correlate with external conquest of a capital or of hub cities.

H5. Intraregional and interregional trade is crucial for city system rise, and economic collapse may be involved in city system collapse. Fluctuations of interregional trade may act either or both to *synchronize* city rise and fall between regions, or to *predict* from city rise and fall in a more developed region that time-delayed rise and fall will occur in a less developed region that is strongly connected by trade. Currency, credit, banking and liquidity are leading indices of development in measuring the interregional impact of trade.

H6. Historical phases that are clearly marked in the population/instability cycles of structural demographic historical dynamics for periods in which agrarian empires that are relatively self-contained, lacking external perturbations (Turchin 2003b, 2005, 2006), should correlate with some but not necessarily all of the phases of city system rise and fall, especially those involving SPI fluctuation. In this context, population growth with low pressure on resources should lead to rises in q and β provided that trade and liquidity allows economic elites to congregate in cities and to provide employment for skilled workers. Peaks of population pressure on resources followed by high SPI levels should precipitate city system declines. In these terms, city system rises and falls:

4a) are likely to be loosely but not strictly coupled into structural demographic variations.

4b) are also interactive with inter-regional competition and global wars (Modelski and Thompson 1996).

4c) are likely to exhibit both longer and shorter (i.e., less predictable) historical periods of stability given this combination of regionally endogenous and regionally exogenous dynamics.

Those hypotheses are testable. The following are more speculative observations about evolutionary tendencies predicated on our findings.

H7. In spite of the growth of gross world product (GWP) rising at faster rates than population, there is no evolutionary historical tendency evident toward either greater stability, greater instability, or alteration in the periodicities of city system ups and downs, in spite of, and probably because of, the pressures of rising global and rising urban populations. Evolutionary stability in city systems, as recovery from small perturbations, apparently remains something to be learned rather than taken for granted.

H8. Stability as recovery from large perturbations through structural demographic oscillatory dynamics, however, has apparently been learned, but urban instabilities are only weakly coupled to oscillatory structural demographic recoveries, so evolutionary learning is only partial. The introduction of new dynamics complicates the question of whether city systems will acquire greater stability

3.3 Scaling Results

Using visual and statistical evidence for changes in the shape parameter, White, Kejžar, Tsallis and Rozenblat (2005) were able in an earlier study to date six Q-periods in Eurasia over the last millennial period. These changes and periods were seen to be related to the framework for studying globalization developed by Modelski and Thompson (1996). In studying multiple regions, a more detailed and dynamical view is taken.

Figure 4 shows the q and β slope parameters fitted by MLE for the 3 regions. Here, a Zipfian tail would have q=1.5 and β=2. The horizontal line shows that this slope and shape is more closely approximated, in the early modern and modern period, for q in China and Europe and for β in Mid-Asia. The figure also shows normalized minima of q and β in which we divide q by 1.5 and β by 2.0 to normalize for the Zipfian.

These results support H1, that the Zipfian is the historical norm both for tails and bodies of city size distributions, approximating β=2 and q=1.5. Mean values for q in the 3 regions vary around q=1.5±.07, consistent with a Zipfian tail, and similarly for variations around β=2 ±.07 for the Pareto slope of the top 10 cities.

Statistical runs tests of whether the variations around the means are random or patterned into larger temporal periods are shown in Table 2 and 3. The runs tests reject the null hypothesis ($p<.01$ for Europe, $p<.05$ for Mid-Asia, and $p<.06$ for China; $p<.00003$ overall), supporting H3 and periods of significant historical variation.

The time periods of successive values above and below the medians represent the rise and fall of q to Zipfian or steeper-than-Zipfian slopes alternating with low-q periods with truncated tails of the distributions. Relatively long city-slump periods occur in the medieval period for all 3 regions, a second slump occurs in Europe in 1450-1500, another in Mid-Asia in 1800-1850, and one in China in 1925 (not shown) when to q falls to 1.02.

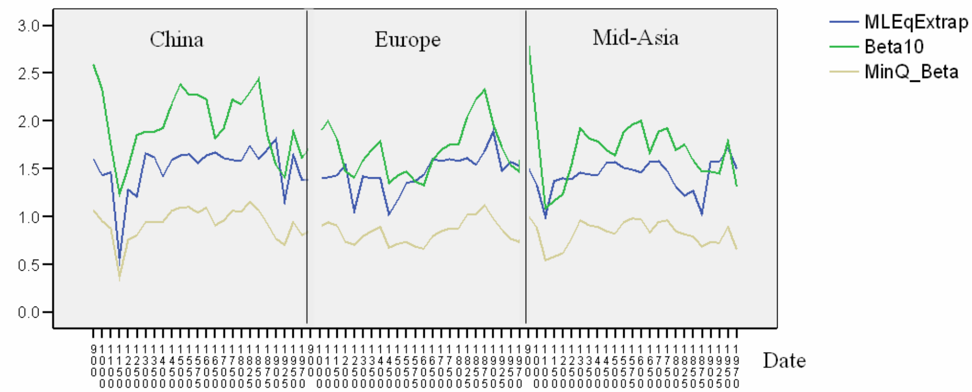

Figure 4: Values of q, β, and their normalized minimum

Table 1: Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Std.Dev/Mean
MLEqChinaExtrap	25	.56	1.81	1.5120	.25475	.16849
MLEEuropeExtrap	23	1.02	1.89	1.4637	.19358	.13225
MLEMidAsIndia	25	1.00	1.72	1.4300	.16763	.11722
BetaTop10China	23	1.23	2.59	1.9744	.35334	.17896
BetaTop10Eur	23	1.33	2.33	1.6971	.27679	.16310
BetaTop10MidAsia	25	1.09	2.86	1.7022	.35392	.20792
MinQ_BetaChina	25	.37	1.16	.9645	.16247	.16845
MinQ_BetaEurope	23	.68	1.26	.9049	.15178	.16773
MinQ_BetaMidAsia	25	.54	1.01	.8252	.13217	.16017
Valid N (listwise)	19					

As shown by the lower dotted line in Figure 5, values of q below 1.3 might be considered as a city system crash or collapse/destruction of primate cities. China and Europe experience an abnormal rise in q in 1900 beyond 1.7 (upper dotted line). This results in a thin tailed distribution (extreme primate cities) that might be considered as a different kind of city system crisis. Some crashes have to do with wars, like the Song loss of their capital to the Jin in 1127. Global wars noted by stars on the lines in Figure 5, might have to do with the punctuations of these periods, but we are unable to evaluate that question statistically. Crashes in q often occur at long intervals (**bold** dates in Table 4), as in Figure 4, with β falling at shorter intervals.

These results support H2 that variations in q and β are *conservative* (since births and normal mortality are slow to affect population measures) but may also *change quickly* in ways consistent with interurban migration, internecine wars and outbreaks of violence or general SPI. They also support H3 that variations in q and β may have long periods of stability, with Zipfian values on both measures correlated with periods of stability and normality, and that stability may be followed by sudden instabilities, or drops in q, β, or both.

As for H6, there are rough correlations for both secular cycles (Turchin 2003b, 2005, 2006, 2007) and Modelski-Thompson (1996) globalization processes with dates of urban crashes, shown in Table 4.

4. Cross-correlation of the Scaling Measures

One of the major patterns of variability in city distributions is the primate city effect: the primate and top ranked cities often form a steeper urban hierarchy in periods of economic boom or empire, or when they are major international trade centers. In periods of decline, they may form a truncated tail compared to the body of the distribution. Further, the slope of the tail of the size distribution (β) tends to change faster than the shape body of the distributions. This is tested using data from all three regions using the autocorrelation function (AFC), where values of a variable in one time period are correlated its values for 1-16 time lags (in this case, 50 years). The upper and lower confidence limits are at 95% for a two-tailed significance test ($p<.05$).The AFC of β compared to q shows a much higher short term continuity (1 lag of 50 years), a recovery period at 5-6 lags, and then autocorrelation largely disappears, while q varies more continuously with more stable long term autocorrelations (up to 16 lags or 800 years). The ratio of q/β has autocorrelation only for Europe, oscillatory and not significant.

Table 2: Runs Tests at medians across

	MLE-q	Beta10	Min(q/1.5, Beta/2)
Test Value(a)	1.51	1.79	.88
Cases < Test Value	35	36	35
Cases >= Test Value	36	37	38
Total Cases	71	73	73
Number of Runs	20	22	22
Z	-3.944	-3.653	-3.645
Asymp. Sig. (2-tailed)	.0001	.0003	.0003

Table 3: Runs Test for temporal variations of *q*

	mle Europe	mle MidAsia	mle China
Test Value(a)	1.43	1.45	1.59
Cases < Test Value	9	11	10
Cases >= Test Value	9	11	12
Total Cases	18	22	22
Number of Runs	4	7	7
Z	-2.673	-1.966	-1.943
Asymp. Sig. (2-tailed)	.008	.049	.052

Table 4: Temporal breaks and urban crashes of β/*q* in the 3 regions

Breaks	950	1150	1430	1640	1850	
Cycle	1	2	3	4	5 (Modelski-Thompson 1996: Table 8.3)	
Mid-Asia:	**1100,**		1450		**1825-75, 1914**	(**major**/minor **urban crashes**)
China:	**1150-1250,**			1650	**1925**	(**major**/minor **urban crashes**)
Europe:		**1250,**	**1450-1500,**		1950?	(**major**/minor **crashes**)

Figure 4 has shown that q and β vary somewhat independently, often correlated positively when $\sigma \ll 1$ ($\kappa \ll 1$, recalling that β is a negative slope and q varies inversely to that slope) but negatively when $\sigma>1$ ($\kappa>1$). So, which affects which over time if the two are synchronously somewhat independent? In time-lagged correlation: Does the shape (q) of the body of the city affect the tail (β) in subsequent periods, or the reverse? β might shape q if long distance trade has an effect on the larger cities engaged in international trade, but q might shape β if it is the waxing and waning of industries in the smaller cities that feed into the export products for the larger cities, as we often see in China and Europe.

What lagged cross-correlations show for China and Europe – but not in Mid-Asia – is that, starting from a maximum correlation at lag 0, high q (e.g., over 1.5) predicts falls in Pareto β over time, reducing the slope of the power law tail below that of the Zipfian. This suggests that high q produces an urban system decline in β. This would contradict a hypothesis of long-distance trade as a driver of rise and fall *in the larger cities*. It would not contradict, however, the possibility that long-distance trade was directly beneficial to the *smaller cities*, in turn affecting the success of the larger cities with a time lag. For China and Europe, where successful long-distance trade was organized on the basis of the diffusion of effective credit mechanisms available to the smaller merchant cities, this seems a plausible explanation for the time-lag findings. These credit mechanisms were not so easily available in Mid-Asia where Islam operated to regulate interest rates to prevent excessive usury.

If there is a correlation between long-distance trade and the rise and fall of population pressure in the secular cycles of agrarian empires, our data might support Turchin's (2007) argument, formulated partly in response to our own studies of the role of trade networks in civilizational dynamics, that it is during the high-pressure (stagflation) period that long-distance trade flourishes. If so, the impact of trade should be reflected first in variations in q, which vary more slowly than β.

The overall pattern in the cross-correlations for the three regions together shows strong correlation synchronically between q and β at lag 0 ($p<.000001$) where high values of q predict falling values of β over time over three 50-year lags. Variables q and β have the least cross-correlate for Mid-Asia, but detailed examination of the Mid-Asia time-lags shows a weak cyclical dynamic of Hi-q→Lo-β →Lo-q→Hi-β that holds to 1950.

5. Historical Network and Interaction Processes

H5 posited that intraregional and interregional trade is crucial for city system rise, and economic collapse may be involved in city system collapse. The data presented here is supportive of hypothesis, but we approach the question first as to whether, at the interregional level, there is synchrony between regions, of what sort, and whether trade is involved in this synchrony.

Turchin (2007) shows evidence for "a great degree of synchrony between the secular cycles in Europe and China during two periods: (1) around the beginning of the Common Era and (2) during the second millennium." We also find evidence for synchrony in city system rise and fall in common temporal variations in q for the second millennium. The correlations in q by time period follow a single factor model, as shown in Table 5, with China contributing the most to the 47% common variance in q between the three regions.

The evidence from city sizes adds detail on dynamical interaction to that of interregional synchrony for the last millennium, supporting H5. Figure 6 shows that changes in q for Mid-Asia lead those of China by 50 years with a hugely significant correlation at 50-year lag 1; Mid-Asia leads Europe by 150 years (lag 3) but the cross-correlation is not quite significant; the cross-correlation for China's q leading Europe by about 100 years (lag 2) is also not quite significant although several earlier estimates of q did show significance (recall that the true values of q may vary somewhat even for unbiased MLE estimates).

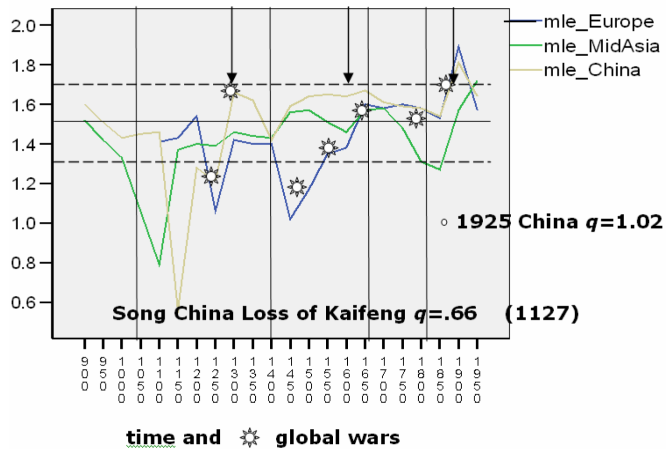

Figure 5: Fitted q parameters for Europe, Mid-Asia, China, 900-1970CE, 50 year lags. Vertical lines show approximate breaks between Turchin's secular cycles for China and Europe Downward arrow: Crises of the 14th, 17th, and 20th Centuries

Table 5: Principal Components: Communalities, Component Matrix, and Variance

	Initial	Extraction
MLEChina	1.000	.660
MLEEurope	1.000	.444
MLEMidAsIndia	1.000	.318

	Component 1
MLEChina	.812
MLEEurope	.667
MLEMidAsIndia	.564

Extraction Method: Principal Component Analysis.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.422	47.403	47.403	1.422	47.403	47.403
2	.944	31.467	78.870			
3	.634	21.130	100.000			

H5 posits historical specificity that Eurasian synchrony has been partly due to trade, particularly that between China and Europe (noting as well that the practice of Islam in Mid-Asian region during this period tended to restrict the full employment of credit mechanisms). The cross-correlation in Figure 8, showing an effect on growth of β in Europe, sustained by the Silk Road trade, for example, suggests that trade is one of the factors causing the growth of power-law tails in urban size distributions, again supporting H5.

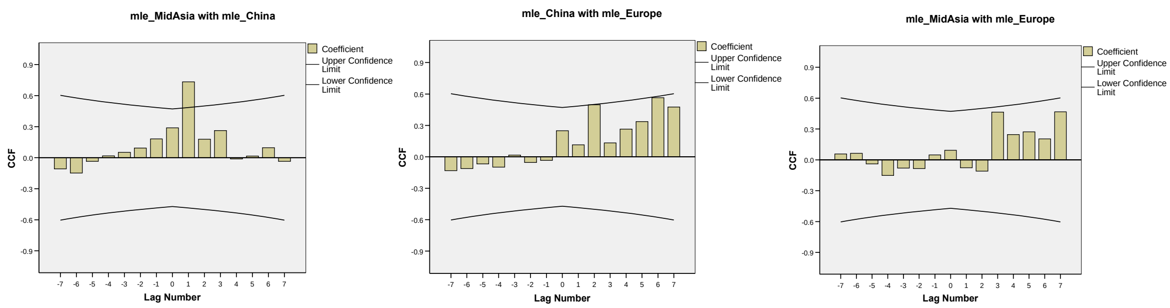

Figure 6: Cross-correlations for temporal effects of one region on another

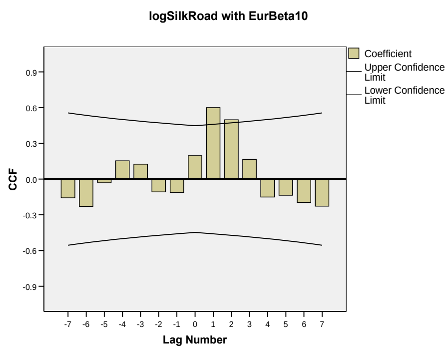

Figure 7: Time-lagged cross-correlation effects of the Silk Road trade on Europe

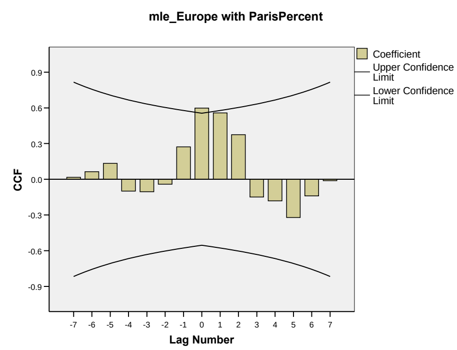

Figure 8: One time-lagged effect of regional q on primate city population

Figure 8 shows, from European data contributed by Turchin, synchrony between higher values of q (normal urban hierarchy) and the percent of French population attracted to Paris as a regional capital and economic center, with this percentage falling after the peak in q.

Our choice of the last millennium to test the interaction of the city size fluctuations with historical dynamics was motivated by the evolution of globalization in Eurasia in this millennium. Key elements in the transition to market-driven globalization occurred in China starting in the period of 10th century invention of national markets, with currencies, banks and market pricing, a historical sequence that leads, through diffusion and competition, to the global system of today (Modelski and Thompson 1996). We found that Chinese economy's credit and liquidity follows closely the rise and fall of q in early Zipfian q~1.5 of Song China until the Jin conquest of Kaifeng, supporting H5. Following the rise, 7-800 years are required, interspersed with long periods of inflation, to regain liquidity and banking favoring international trade. Rise in the liquidity graph (not shown) correlates with the first modern Chinese bank, the Rishengchang (est. 1824). It grew to operate in every major Chinese city, folding in bankruptcy in 1932.

For further tests of H6, we have scant data on total population relative to resources, and we have reliable data for the last millennium only for England in comparison with our Eurasian city data. There are few points of comparison, but temporal synchronies appear in those few points: 1300 and 1625 are the peaks of scarcity for ratios of people to resources and pre-1100, 1450, and 1750 are the troughs of plentiful resources. The peaks correspond to slumps in q and the troughs to rises. It is impossible to rule out at this point the possibility suggested in H6 that the urban system fluctuations that we observe are interactively linked to Turchin's secular cycles, particularly if we include both types of fluctuations: those in q, in β, and in our normalized minimum of the two, as wall, which reflects either type of slump.

Testing H6 against sociopolitical instability (SPI), J. S. Lee's data on Chinese internecine wars, reproduced in Figure 1, was interpreted by Lee as showing 800-year cycles of internecine conflict, weakly separated into two 400-year periods. These SPI fluctuations resemble China's the 800 year lag between urban system collapses in q with a dip in β in the middle of this period that might indicate shorter major fluctuations of the Chinese city system.[10] Mid-Asia also shows 7-800 year lags in our data between urban system collapses, and

[10] In some of our earlier analyses there were tenuous indicators of 400-year periods and possibly 200-year periods of city size oscillation that seemed to correspond more closely with Turchin's secular cycles. With more accurate MLE measures, however, we see longer periods of

also has some dips in β within the longer period. There is some support here for H6 for coupling between urban system dynamics and structural demographic historical cycles.

Strong evidence for the coupling of urban system dynamics with population/instability (structural demography) historical dynamics is presented in Figure 10. Much as SPI leads population declines in historical dynamics, the top graph in the figure shows that SPI is synchronously correlated with low β (reduced urban hierarchy) in city distributions for China. β recovers following peaks in SPI, just as population does. The bottom graph in Figure 10 shows a 50 year lag between a high q/β ratio and rise in SPI, much as population growth relative to resources predicts rise in SPI.

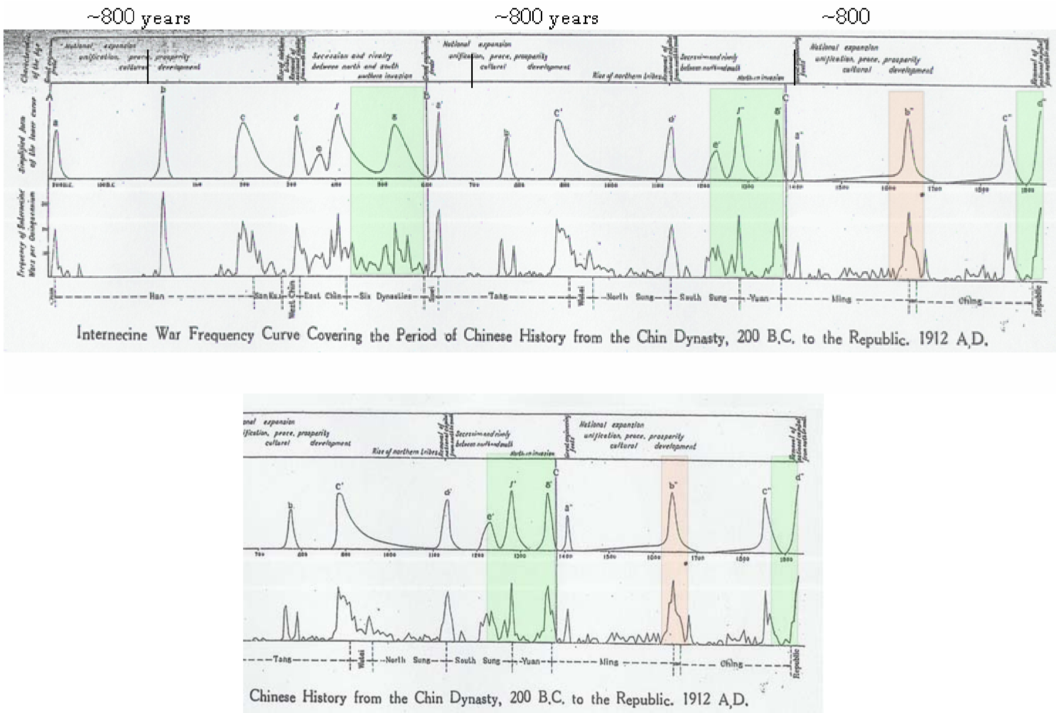

Figure 9: Long-cycle internecine war cycles in China

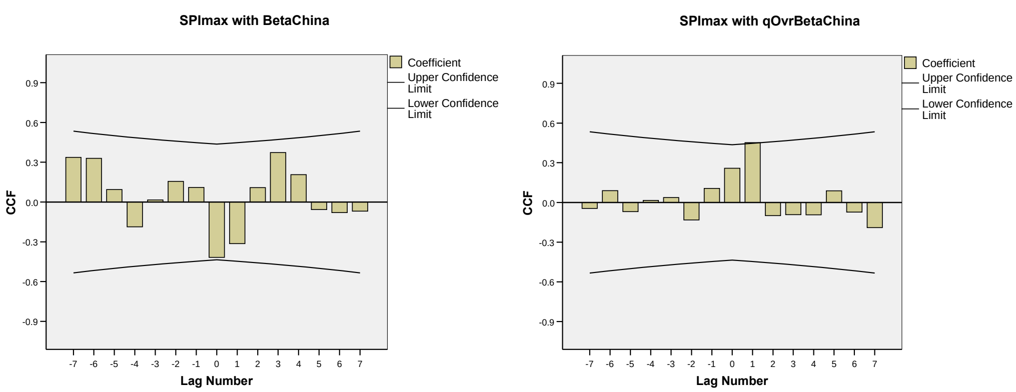

Figure 10: Cross-correlations of q and β and sociopolitical instability (SPI)

6. Conclusions

This study depends on reliable unbiased estimates of variations in city size distributions using Maximum likelihood (MLE) that allowed adequate precision, for longitudinal comparisons even with small samples. It is built from two sets of sources: mathematical/measurement concepts incorporating city size distribution and the

stability, and we attribute the appearance of shorter oscillations in the earlier analysis to biased estimations that, in introducing intermittent error, tended to break longer periods of stability into what appeared to be shorter ones.

quantitative history work of Goldstone (1991), Nefedov (1999), Turchin (2003b), and Spufford (2002)

By focusing on the 75 largest cities over a antiquely starting series of time periods that spaced enough with quantitative variations of the full cycles of city system oscillations, Chandler (1987) provided usable data for studying how city system evolutions couple with agrarian sociopolitical dynamics, both for the early period of globalization, where China had the largest number of large cities,[11] and for later periods.

We did find strong evidence of historical periods of rise and fall in the city systems of different regions, and time lagged effects of changes in city size distributions in one region on other regions. These are weak and slow from Mid-Asia to China, and strong and fast from China to Europe, which makes sense in terms of the Silk Roads trade. This provides additional evidence of synchronies missing from Chase-Dunn, Niemeyer, Alvarez, Inoue, Lawrence and Carlson (2006) and the studies of Eurasian synchrony. Many of the correlations, however, are time-lagged rather than temporally synchronous. The effects run in the directions suggested by Modelski and Thompson (1996).

We are reasonably confident in concluding that the Pareto I and II (*q*-exponential) measures of city hierarchies through time, especially when used in combination, can provide a measurement paradigm of standardized methods and tests of replication in historical comparisons. The attractive features of the *q*-measure gained added benefit from the precision of our measures with the use of MLE.

Richness of supporting data would logically take us next to Middle Asia, its subregions, and the larger world from 700 CE that embraced the rise of Islam, the Mongol use of the Silk Roads and development of new towns and cities on those routes to link China and the rest of Middle Asia into a global system. Such a study, modeled on this one, would include the role of the Indic subcontinent, and that of the Mongols (Barfield 1989, Boyle 1977) in trade and conquest, the Arab colonization of North Africa and Spain, and the feeding of urban developments in the Mediterranean, Russia, and Europe.

Comparing our results, measurements, and mathematical models to those of the structural demography or secular cycles studies of Goldstone (1991), Nefedov (1999), and Turchin (2003b, 2005), we find several novelties that separate our findings from Turchin's (2003b:12-13) two equation "secular cycle" oscillation model for historical fluctuations. These are somewhat analogous to the Turchin (2004:257-258) predator-prey model, where "If predators are at the low point (P1), prey will increase, but if predator numbers are high (P2), prey numbers will crash. The full model for the system will have two equations, one for prey and one for predators, and we know that such two dimensional models are perfectly capable of displaying cyclic behaviour." Aalthough varying in process time between cycles, this dynamic works optimally when one of the interactive variables (say, population/resource ratio measure of scarcity and sociopolitical violence) is offset by ¼ cycle. Our cycle of city-size oscillations might be 2 and sometimes 4 times as long as Turchin's secular cycles.[12] It is possible that the city cycle operates at one or both of these time-scales, and at the spatial scales of larger alternating civilizational networks of states and forms of empire. Long city-size system oscillations of ca. 800 years would not be offset by a ¼ cycle but by 1/8th of a cycle, which is a long period of instability (vulnerable to conquest from the outside following internal instabilities). From our perspective, however, sociopolitical instability is not smoothly cyclical but episodic. Rebellions, insurrections, and all sorts of protest are events that mobilize people in a given time and generation, and that impacts that, when repeated frequently, have massive effects. We see this in long-term correlations with SPI, such as internecine wars in China.

There are effects of trade fluctuations (to which we might add effects of network structure in trade as it operates on particular cities and regions) in these models, particularly those of the monetary liquidity variable for China and the effects of the trade-related Silk Roads variable on *q*. It might be possible to reconstruct trade routes as historical time series with their trade volumes' ordinal ranking. These strong effects, along with disruptive conflicts and political or empire boundaries on the economies of individual cities and regions, would show dynamical interactions in secular cycle and urban systems' rise and fall.

Although forward-looking prediction is not done, we hypothesize that MLE delivers reliable estimation of city-size distribution oscillations. Our data concerning globalizing modernization are consistent with prior knowledge and but given developmental trends of scale – larger global cities, larger total urban population, and larger total population – the extent of oscillations and instabilities are also startling. Our scaling also allows investigation of where the crossover occurs (Pareto II "scale" or σ) as the tail of the distribution approaches

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[11] Future refinement of the Chinese data for comparison with other regions will consider whether to adjust for Chandler's possible underestimation of walled city populations, but his one biasing assumption for China's walled cities of underestimating density does not carry over to other regions.

[12] J.S. Lee divides his 800 year periods into two periods of 400, seeming to do with an early growth of early forms of "empire" in a region, then a time of turbulence in the second period; then a new cycle of empire

power law. The crossover estimate also gives the possibility of estimating total urban population.

What is startling is that there are some very long-wave oscillations in *q*. Hopefully, a long-term trend toward the contemporary structure of roughly Zipfian city distributions is an indicator of stability. However, the 20th century data still indicate that instabilities are still very much present and thus likely to rest on historical contingencies (somewhat like the occurrence of a next earthquake larger than any seen in *x* years prior) and very much open to the effects of warfare and internal conflicts that are likely to be affected by population growth as opposed to stabilization of trade benefiting per-capita-resources ratios.

Our results allow us to consider the edge-of-chaos metaphor of complexity with respect of *q* as a first, even if insufficient approximation, to an explanation for what we see historically. It is a truism to say that complexity, life, history, and complex systems generally stand somewhere between rigidity at one pole, which might be exemplified by $q>1.7$, and on the other, an exponential random distribution ($q\approx1$) of city sizes, or the heightened unpredictability of chaos ($0<q\ll1$).[13] But we do not see support for *equilibrium* on the "edge of chaos" in these data. The historical *q*-periods of China and other regions tend to cluster, somewhat like "edging on chaos," near an average of unstable $q\approx1.5$, but with oscillations not far from equilibrium. From that average they may fall into decentralized chaos, here in the metaphoric sense (but with exceptionally low *q*) in which the power-law tails are absent (with larger cities seemingly crushed in size by internecine wars) and smaller cities are frequent relative to hubs, or rise into regimes are affected by massive external drains on the economy or political policies that seem to put *q* into abnormally rigid states (exceptionally high *q*). The directions of change in *q* are largely predictable as a function of the current-state variables (such as population/resource ratios and sociopolitical violence) in the historical dynamics models up to, but not yet including, the contemporary period. How to derive predictions for the contemporary era is not yet evident given the new configurations of industrial societies, but it is very probable that such predictions as do emerge for the present will contain processes operative in the past.

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[13] Technically, the mathematics assigned to chaos is a deterministic departure from randomness in which a dynamic trajectory never settles down into equilibrium, and small differences in initial conditions lead to divergent trajectories. The link between empirical history and "edge of chaos" is typically done by simulation.

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