Political Equilibria in a Stochastic Valence Model of Elections in Turkey.*

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Abstract

Models of elections tend to predict that parties will maximize votes by converging to an electoral center. There is no empirical support for this prediction. In order to account for the phenomenon of political divergence, this paper offers a stochastic electoral model where party leaders or candidates are differentiated by differing valences—the electoral perception of the quality of the party leader. If valence is simply intrinsic, then it can be shown that there is a “convergence coefficient”, defined in terms of the empirical parameters, that must be bounded above by the dimension of the space, in order for the electoral mean to be a Nash equilibrium. This model is applied to elections in Turkey in 1999 and 2002.

The idea of valence is then extended to include the possibility that activist groups contribute resources to their favored parties in response to policy concessions from the parties. The equilibrium result is that parties, in order to maximize vote share, must balance a centripetal electoral force against a centrifugal activist effect. We estimate pure spatial models and models with sociodemographic valences, and use simulations to compare the equilibrium predictions with the estimated party positions.

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1 Introduction: Modeling Popular Support

The early work in modeling elections focused on two-party competition, and assumed a one-dimensional policy space, $X$, and “deterministic” voter choice. The models showed the existence of a “core” point, unbeaten under majority rule vote, at the median of the electoral distribution. Such models implied that there would be strong centripetal political forces causing parties to converge to the electoral center (Hotelling, 1929; Downs, 1957). In higher dimensions, such two party “pure strategy Nash equilibria” (PNE) generally do not exist, so the theory did not cover empirical situations where two or more policy dimensions were relevant.1 It has been shown, however, that there would exist mixed strategy Nash equilibria whose support lies within a subset of the policy space known as the “uncovered set.”2 “Attractors” of the political process, such as the “core”, the “uncovered set” or the “heart” (Schofield, 1999) are centrally located with respect to the distribution of voters’ ideal points. The theoretical prediction that political candidates converge to the center is very much at odds with empirical evidence from U.S. presidential elections that political candidates do not locate themselves close to the electoral center.3

The deterministic electoral model is also ill-suited to deal with the multiparty case. (Here multiparty refers to the situation where the number of candidates or parties, $p$, is at least three.) As a result, recent work has focused on “stochastic” models which are, in principle, compatible with empirical models of voter choice.4 In such models, the behavior of each voter is modeled by a vector of choice probabilities. Various theoretical results for this class of models suggested that vote maximizing parties would converge to the mean of the electoral distribution of voter ideal points.5

Empirical estimates of party positions in European multiparty polities can be constructed on the basis of various techniques of content analysis of party manifestos.6 More recent analyses have been based on factor analysis of electoral survey data to obtain a multidimensional description of the main political issues in various countries. All these empirical analyses have obtained policy spaces that are two dimensional. These techniques allow for the estimation of the positions of the parties in the empirically inferred policy space. These estimates have found no general tendency for parties to converge to the center.7

The various empirical electoral models can be combined with simulation techniques to determine how parties should respond to electoral incentives in order to maximize their vote shares. Schofield and Sened (2006), in their simulation of elections in Israel in the period 1988 to 1996, found that vote maximizing

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1See Saari (1997) and the survey in Austen-Smith and Banks 1999.
2Banks, Duggan and Le Breton 2006.
3Poole and Rosenthal 1984; Schofield, Miller and Martin 2003. See also the empirical work in the companion paper by Schofield, Claassen, Ozdemir and Zakharov (2009a).
5Hinich 1977; Lin, Enelow and Dorussen 1999; Banks and Duggan 2005; McKelvey and Patty 2006.
7See Adams and Merrill 1999, for example.
parties did not converge to the electoral origin. It may be objected that factor analysis of survey data gives only a crude estimate of the variation in voter preferences, while vote maximization disregards the complex incentives that parties face. Nonetheless, as a modeling exercise, the stochastic model for Israel seemed to provide a plausible account of the nature of individual choice as well as the party positioning decision. Although the simulated equilibrium positions of the parties in Israel were not identical to the estimated positions, the positions were generally far from the origin, and for some of the parties very close to their estimated positions. The purpose of this paper is to attempt to extend the stochastic empirical model so as to close the apparent disparity between the simulated equilibrium positions of the parties, and the estimated positions.

The key to the contradiction between the non-convergence result of Schofield and Sened, and the convergence result in other work on the formal stochastic model was the incorporation of an asymmetry in the perception of the quality of the party leaders, expressed in terms of valence (Stokes, 1992). Stokes (1963: 373) used the term valence issues to refer to those that “involve the linking of the parties with some condition that is positively or negatively valued by the electorate.” As he observes, “in American presidential elections... it is remarkable how many valence issues have held the center of the stage.” The notion of valence has formed the basis for a recent extensive analyses of British, Canadian and US electoral response by Clarke et al. (2009 a,b). They argue that electoral responses in Britain were a reflection largely of [the] changing perceptions of the decision-making competence of the main political parties and their leaders. At any point in time, [the] preferences were strongly influenced by their perceptions of the capacity of the rival parties—the putative alternative governments of the day—to solve the major policy problems facing the country.

In the model presented here, the average weight given to the perceived quality of the leader of the \( j \)th party is called the party’s intrinsic valence. In empirical models this valence is assumed to be exogenous, so it is independent of the party’s position. The valence coefficients for each party are generated by the estimation of the stochastic model, based on the “multinomial logit” (MNL) assumption that the stochastic errors have a “Type I extreme value or Gumbel distribution” (Dow and Endersby, 2004). These valence terms add to the statistical significance of the model. In general, valence reflects the overall degree to which the party is perceived to have shown itself able to govern effectively in the past, or is likely to be able to govern well in the future.\(^9\)

The Appendix considers a pure spatial stochastic vote model, with party specific intrinsic valences, based on the same distribution assumption, and on the

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\(^8\)Over 60% of the individual votes were correctly modeled.

\(^9\)See Penn (2009). Notice, however, that valence refers to the perception by voters of the quality of political leaders. Recent work by Westen (2007) suggests this perception need have no “rational” basis. However, related work (Schofield et al., 2009a,b) shows that voter perception of character traits has a strong effect on candidate positions in the United States.
assumption that each party leader attempts to maximize the party’s vote share. Results from Schofield (2007a,b) give the necessary and sufficient conditions under which there is a “local pure strategy Nash equilibrium” (LNE) of this model at the joint electoral mean (that is, where each party adopts the same position, \( z_0 \), at the mean of the electoral distribution). Theorem 2 in the Appendix shows that a “convergence coefficient”, \( c \), incorporating all the parameters of the model, can be defined. This coefficient, \( c \), involves the differences in the valences of the party leaders, and the “spatial coefficient” \( \beta \). When the policy space, \( X \), is assumed to be of dimension \( w \), then the necessary condition for existence of an LNE at the electoral center\(^{10} \) is that the coefficient, \( c \), is bounded above by \( w \).

When the necessary condition fails, then parties, in equilibrium, will adopt divergent positions. Because a pure strategy Nash equilibrium must be a local equilibrium, the failure of existence of LNE when all parties are at the electoral mean implies non existence of such a centrist PNE. In this case, a party whose leader has the lowest valence will have the greatest electoral incentive to move away from the electoral mean.\(^{11} \) Other low valence parties will follow suit, and the local equilibrium will be one where parties are distributed along a “principal electoral axis.”\(^{12} \) The general conclusion is that, with all other parameters fixed, then a convergent LNE can be guaranteed only when \( \beta \) is “sufficiently” small. Thus, divergence away from the electoral mean becomes more likely the greater are \( \beta \), the valence differences and the variance of the electoral distribution.\(^{13} \)

The innovation of this paper is that in addition to intrinsic valence, we also incorporate “sociodemographic valence.” These party specific valence terms are associated with different groups in the society, and are defined by dichotomous or continuous characteristics of different subgroups in the population. We apply this valence model by considering in some detail a sequence of elections in Turkey from 1999 to 2007. The election results are given in Tables 1, 2, and 3, which also provide the acronyms for the various parties.

[Insert Tables 1, 2 and 3 and Figures 1 and 2 here].

As in other related work, the empirical models were based on factor analyses of voter surveys.\(^{14} \) Figures 1 and 2 show the electoral distributions (based on sample surveys of sizes 635 and 483 respectively) and estimates of party positions for 1999 and 2002.\(^{15} \) The two dimensions in both years were a “left-

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\(^{10} \) Again, the electoral center, or origin, is defined to be the mean of the distribution of voter ideal points.

\(^{11} \) This follows for theoretical reasons as shown in Schofield (2007a). When \( c > w \) the positive eigenvalue of the Hessian of the vote share function of a low valence party will be large and positive at the origin. This implies that its vote share increases rapidly as the party moves away from the origin. Simulation of empirical models for Israel (Schofield and Sened, 2006) came to the same conclusion.

\(^{12} \) The principal electoral axis is defined to be the one dimensional subspace along which the variance of the distribution of voter ideal points is maximum.

\(^{13} \) These results are presented for the reader’s convenience in the context of the more general model described in the Appendix.

\(^{14} \) The estimations presented below are based on factor analyses of sample surveys conducted by Veri Arastima for TUSES.

\(^{15} \) The party positions were estimated using expert analyses, in the same way as the work
right” religion axis and a “north-south” Nationalism axis, with secularism or “Kemalism” on the left and Turkish nationalism to the north. (See also Çarkoğlu and Hinich (2006) for a spatial model of the 1999 election).

Minor differences between these two figures include the disappearance of the Virtue Party (FP) which was banned by the Constitutional Court in 2001, and the change of the name of the pro-Kurdish party from HADEP to DEHAP. The most important change is the appearance of the new Justice and Development Party (AKP) in 2002, essentially substituting for the outlawed Virtue Party.

In 1999, a DSP minority government formed, supported by ANAP and DYP. This only lasted about 4 months, and was replaced by a DSP-ANAP-MHP coalition, indicating the difficulty of negotiating a coalition compromise across the disparate policy positions of the coalition members. During the period 1999–2002, Turkey experienced two severe economic crises. As Tables 1 and 2 show, the vote shares of the parties in the governing coalition went from about 53 percent in 1999 to less than 15 percent in 2002. In 2002, a 10% cut-off rule was instituted. As Table 3 makes clear, seven parties obtained less than 10% of the vote in 2002, and won no seats. The AKP won 34% of the vote, but because of the cut-off rule, it obtained a majority of the seats (363 out of 550). In 2007, the AKP did even better, taking about 46% of the vote, against 21% for the CHP. The Kurdish Freedom and Solidarity Party avoided the 10% cut-off rule, by contesting the elections as independent non-party candidates, winning 24 seats with less that 5% of the vote.

The point of this example is that a comparison of Figures 1 and 2 suggest that there was very little change in policy positions of the parties between 1999 and 2002. The basis of support for the AKP may be regarded as a similar to that of the banned FP, which suggests that the leader of this party changed the party’s policy position on the religion axis, adopting a much less radical position.

In sum, the standard spatial model is unable to explain the change in the electoral outcome, taken together with the relative unchanged positioning of the parties between 1999 and 2002.

Section 2 of the paper considers the details of the multinomial logit (MNL) model for Turkey for 1999 and 2002. In particular, this section shows that the pure spatial model with intrinsic valence predicts that the parties diverge away from the origin. To illustrate, Table 5 shows that the lowest valence party in 2002 was the Motherland Party (ANAP) while the Republican People’s Party (CHP) had the highest valence. The convergence coefficient was computed to be 5.94, far greater than the upper bound of 2. Figure 3 presents an estimate of one of the LNE obtained from simulation of vote maximizing behavior of the parties, under the assumption of the pure spatial model with intrinsic valence. As expected from the theoretical result, the LNE is non centrist. Note however,
The LNE positions for the pure spatial model given in Figure 3 are quite different from the estimated positions in Figure 2.

The formal model with intrinsic valence was extended to include both sociodemographic variables as well as the influence of party activists. Sociodemographic variables can be interpreted as specific valences associated with different sociodemographic subgroups of the electorate. We show that these sociodemographic valences can be used to estimate the influences of party activists. Theorem 1 in the Appendix\textsuperscript{17} gives the first order balance condition for local equilibrium in the stochastic electoral model involving sociodemographic valences and activists in dimension $w$. The condition requires the balancing of a centrifugal marginal activist pull (or gradient) against a marginal electoral pull. In general, if the intrinsic valence of a party leader falls, then the marginal electoral pull also falls, so balance requires that the leader adopt a position closer to the preferred position of the party activists.

The pure spatial model, with intrinsic valences, and a joint model, with sociodemographic valences, but without activists, are compared using simulation to determine the LNE in these models. This allows us to determine which model better explains the party positions. For example, Figure 4 shows the LNE based on a joint sociodemographic model for 2002. In this figure, the LNE position for the Kurdish party, HADEP, is a consequence of the high electoral pull by Kurdish voters located in the lower left of the figure. Similarly, the position of the CHP on the left of the figure is estimated to be due to the electoral pull by Alevi voters who are Shia, rather than Sunni and can be regarder as supporters of the secular state. Although Figure 4 gives a superior prediction of the party positions than Figure 3, there is still a discrepancy between the estimated positions of Figure 2 and the LNE in Figure 4. We argue that the difference between these two vectors of party positions, as presented in Figures 2 and 4, can be used to provide an estimation of the marginal activist pulls influencing the parties.

More generally, we suggest that the combined model, with sociodemographic variables and activists, can be used as a tool with which to study the political configuration of such a complex society. In the conclusion we suggest that the full model involving activists may be applicable to the study of what Epstein et al (2006) call “partial democracies”, where a political leader must maintain popular support, not just by winning elections, but by maintaining the allegiance of powerful activist groups in the society.

\textsuperscript{17}The results in the Appendix extend the version of the activist model originally proposed by Aldrich 1983 and developed in Schofield 2006a.
2 Elections in Turkey 1999-2007

The Appendix defines an empirical electoral model, denoted \( M(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta; \Psi; \mathbf{V}) \) which utilizes socio-demographic variables, denoted \( \boldsymbol{\theta} \).

The symbol, \( \mathbf{V} \), denotes a family of egalitarian vote functions, one for each party, and under which all voters are counted equally. The formal model of the Appendix considers a more general class of vote functions where the voters vary in their weights, thus allowing for complex electoral rules. In the Appendix, the egalitarian family is denoted \( \mathbf{V}_e \). The symbol, \( \Psi \), denotes the Gumbel stochastic distribution on the errors. To simplify notation in the applications that follow we delete reference to \( \mathbf{V} \) and \( \Psi \).

This empirical model assumes that the utility function of voter \( i \) is given by the expression

\[
u_{ij}(x_i, z_j) = \Lambda_j + (\theta_j \cdot \eta_i) - \beta\|x_i - z_j\|^2 + \varepsilon_j,
\]

Here, the spatial coefficient is denoted \( \beta \) and \( \boldsymbol{\Lambda} = \{\Lambda_j : j \in P\} \) are the intrinsic valences (relative to a baseline party, \( k^* \)).\(^{18}\) The relative valence, \( \Lambda_j \), gives the average belief of the voters in the electorate concerning the quality of the leader of party \( j \) in comparison to the leader of the baseline party, \( k^* \). The symbol, \( \theta \), denotes a set of \( m \)-vectors \( \{\theta_j\} \) representing the effect of the \( m \) different socio-demographic parameters (class, domicile, education, income, religious orientation, etc.) on the beliefs of the various subgroups in the polity on the competence of party \( j \). The symbol \( \eta_i \) is an \( m \)-vector denoting the \( i^{th} \) individual’s relevant “socio-demographic” characteristics. The composition \((\theta_j \cdot \eta_i)\) is the scalar product and can be interpreted as the group specific valence ascribed to party \( j \) as a consequence of the various socio-demographic characteristics of voter \( i \). Again, these socio-demographic variables will be normalized with respect to the baseline party \( k^* \), essentially by estimating \((\theta_j - \theta_{k^*}) \cdot \eta_i\). This scalar term is called the total socio-demographic valence of voter \( i \) for party \( j \). The \( t^{th} \) term in this scalar is called the socio-demographic valence of \( i \) as a result of membership by \( i \) of the \( t^{th} \) group, or, more briefly, the \( t^{th} \) group specific socio-demographic valence for the leader of party \( j \).

The vector \( \mathbf{z} = (z_1, \ldots, z_p) \in X^p \) is the set of party positions, while \( \mathbf{x} = (x_1, \ldots, x_n) \in X^n \) is the set of ideal points of the voters in \( N \). When \( \beta \) is assumed zero then the model is called pure socio-demographic (SD), and denoted \( M(\boldsymbol{\Lambda}, \boldsymbol{\theta}) \). When \( \{\theta_j\} \) are all assumed zero then the model is called pure spatial, and denoted \( M(\boldsymbol{\Lambda}, \beta) \). The full empirical model, \( M(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta) \), is called joint. The socio-demographic variables allow us to incorporate characteristics common to

\(^{18}\)Note that in the empirical models discussed below, these are specified relative to the baseline party, the DYP.

\(^{19}\)For example, in Table 6 and 7 there are 6 socio-demographic variables, so \( m = 6 \). An individual who is Alevi has \( \eta_{Alevi} = 1 \). The coefficient for the CHP party for an Alevi is 3.089 in 1999, and this is the group-specific valence that a voter who is a member of the group of Alevi voters has for this party. Note again that this is specified relative to the baseline party, the DYP. These valences may be the result of the perception of the leader’s ability, as displayed in the past, or of the particular partiality of these voters to choose the party, independently of the party’s policy position.
specific groups of supporters of any party. Not accounting for these characteristics in the analysis will bias the estimates of the intrinsic valences of the parties.

Tables 4 and 5 give the details of the pure spatial MNL models for the elections of 1999 and 2002 in Turkey, while Tables 6 and 7 give the details of the joint MNL models. The differences in log marginal likelihoods for the three different models then gives the log Bayes’ factor for the pairwise comparisons.\textsuperscript{20} The log Bayes’ factors show that the joint and pure spatial MNL models were clearly superior to the SD models. In addition the joint models were superior to the pure spatial models.\textsuperscript{21} We can infer that, though the sociodemographic variables are useful, by themselves they do not give an accurate model of voter choice.\textsuperscript{22} It is necessary to combine the pure spatial model, including the valence terms, with the sociodemographic valences to obtain a superior estimation of voter choice.

Comparing Tables 4 and 5, it is clear that the relative valences of the ANAP and MHP, under the pure spatial model, dropped between 1999 and 2002. In 1999, the estimated $\Lambda_{ANAP}$ was $+0.336$, was significantly different from zero, whereas in 2002 it was $-0.31$ (and thus not significantly different from zero).\textsuperscript{23} Similarly $\Lambda_{MHP}$ fell from a significant value of $+0.666$ in 1999 to $-0.12$ in 2002. The estimated relative valence, $\Lambda_{AKP}$, of the new Justice and Development Party (AKP) in 2002 was $+0.78$, in comparison to the valence of the FP of $-0.159$ in 1999. Since the AKP can be regarded as a transformed FP, under the leadership of Recep Tayyip Erdogan, we can infer from the confidence intervals on these two relative valences that this was a significant change due to Erdogan’s leadership.\textsuperscript{24}

It should be noted that the $\beta$ coefficients for the pure spatial models were 0.375 in 1999, and 1.52 in 2002. Both of these are estimated to be non-zero at the 0.001 level. Indeed, they are significantly different from each other,\textsuperscript{25} suggesting that electoral preferences over policy had become more intense.

We first use the results of the formal pure spatial model given in the Appendix to compute estimates of the convergence coefficients. These computations suggest that convergence to an electoral center is not to be expected in these elections. We then use simulation to determine the LNE of the empirical joint models, again showing non-convergence. This allows us to obtain information

\textsuperscript{20}Since the Bayes’ factor (Kass and Raftery, 1995) for a comparison of two models is simply the ratio of marginal likelihoods, the log Bayes’ factor is the difference in log likelihoods.

\textsuperscript{21}The log Bayes factors for the joint models over the sociodemographic models were highly significant at +31 in 1999 and +58 in 2002. The Bayes’ factors for the joint over the spatial models were also significant, and estimated to be +6 and +5 in 1999 and 2002, respectively.

\textsuperscript{22}Sociodemographic models are standard in the empirical voting literature.

\textsuperscript{23}These tables show the standard errors of the coefficients, as well as the t-values, the ratios of the estimated coefficient to the standard error.

\textsuperscript{24}Although Erdogan was the party leader, Abdullah Gul became Prime Minister after the November 2002 election because Erdogan was banned from holding office. Erdogan took over as Prime Minister after winning a by-election in March 2003.

\textsuperscript{25}The 95% confidence interval for $\beta_{1999}$ is $[0.2,0.55]$ and for $\beta_{2002}$ it is $[1.28,1.76]$.
about activist support for the parties.

2.1 The 2002 Election

Figure 3 shows the smoothed estimate of the voter ideal points in 2002. This distribution gives the 2 by 2 voter covariance matrix, with an electoral variance on the first axis (religion) estimated to be 1.18 while the electoral variance on the second axis (nationalism) was 1.15. The total electoral variance was $\sigma^2 = 2.33$, with an electoral standard deviation of $\sigma = 1.52$ The covariance between the two axes was equal to 0.74.

Thus the voter covariance matrix is

$$\nabla_0 = \begin{bmatrix} 1.18 & 0.74 \\ 0.74 & 1.15 \end{bmatrix}$$

with $\text{trace}(\nabla_0) = 2.33$.

The eigenvalues of this matrix are 1.9, with major eigenvector $(+1.0,+0.97)$ and 0.43, with minor eigenvector $(-0.97,+1.0)$. The major eigenvector corresponds to the principal electoral axis, aligned at approximately 45 degrees to the religion axis.

For the pure spatial model $M(\Lambda, \beta)$, the $\beta$ coefficient was 1.52. The valence terms are estimated in contrast with the valence of the DYP, and the the party with the lowest relative valence is ANAP with $\Lambda_{ANAP} = -0.31$. By definition, $\Lambda_{DYP} = 0$. The vector of relative valences is then

$$(\Lambda_{ANAP}, \Lambda_{MHP}, \Lambda_{DYP}, \Lambda_{HADEP}, \Lambda_{AKP}, \Lambda_{CHP}) = (-0.31, -0.12, 0.0, 0.43, 0.78, 1.33).$$

When all parties are at the origin, the probability, $\rho_{ANAP}$, that a voter chooses ANAP in the model $M(\Lambda, \beta)$, is independent of the voter. The Appendix, equation (7), shows that this is given by

$$\exp(-0.31)$$

$$\frac{\exp(-0.31) + \exp(-0.12) + \exp(0.0) + \exp(0.43) + \exp(0.78) + \exp(1.33)}{1 + \exp(0.19) + \exp(0.31) + \exp(0.74) + \exp(1.09) + \exp(1.64)}^{-1}$$

$$= [1 + 1.2 + 1.36 + 2.09 + 2.97 + 3.2]^{-1}$$

$$= 0.08.$$

Below, we show that the 95% confidence interval on $\rho_{ANAP}$ is $[0.05, 0.11]$, which includes the actual vote share (5.13%) in 2002. The Appendix shows that the Hessian of the vote share function of ANAP, when
all parties are at the origin, is given by the characteristic matrix of ANAP:

\[
C_{ANAP} = 2\beta(1 - 2\rho_{ANAP})\nabla_0 - I \\
= 2 \times (1.52) \times [(1 - (2 \times 0.08))]\nabla_0 - I \\
= (2.55) \begin{bmatrix}
1.18 & 0.74 \\
0.74 & 1.15
\end{bmatrix} - I \\
= \begin{bmatrix}
2.01 & 1.88 \\
1.88 & 1.93
\end{bmatrix}.
\]

Moreover, the convergence coefficient,

\[
c = 2\beta(1 - 2\rho_{ANAP}) \text{trace}(\nabla_0) = 2.55 \times 2.33 = 5.94.
\]

This greatly exceeds the upper bound of +2.0 for convergence to the electoral origin. The major eigenvalue for the ANAP characteristic matrix is +3.85, with eigenvector (+1.0, +0.98), while the minor eigenvalue is +0.09, with orthogonal, minor eigenvector (−0.98, +1.0). The eigenvectors of this Hessian are almost perfectly aligned with the principal and minor components, or axes, of the electoral distribution.

Although the electoral origin satisfies the first order condition for local equilibrium, it follows from a standard result that the electoral origin is a minimum of the vote share function of ANAP, when the other parties are at the same position. On both principal and minor axes, the vote share of ANAP increases as it moves away from the electoral origin, but because the major eigenvalue is much larger than the minor one, we can expect that the AKP (as well the other parties) in equilibrium to adopt positions along a single eigenvector. Figure 4 presents one of a number of LNE obtained from simulation of the pure spatial model. Note that all the positions in this LNE lie on the principal axis given by the eigenvector (1.0, 1.0). Note also that these equilibrium positions are far from the estimated positions of the parties, as shown in Figure 3.

The Appendix shows that the standard error on \(\Lambda_{ANAP}\) is \(h = 0.19\), so

\[
\rho_{ANAP}(\Lambda_{ANAP} + h) = \rho_{ANAP}(\Lambda_{ANAP}) + h \frac{d\rho_{ANAP}}{d\Lambda} \\
= \rho_{ANAP}(\Lambda_{ANAP}) + h \rho_{ANAP}(1 - \rho_{ANAP}).
\]

This gives a standard error of 0.014 and a 95% confidence interval on \(\rho_{ANAP}\) of [0.05, 0.11]. Since the standard error on \(\beta\) is 0.12, giving a confidence interval on \(\beta\) of approximately [1.28, 1.76], the standard error on \(c\) is 0.27. Using the lower bound on \(\beta\) and upper bound on \(\rho_{ANAP}\) gives an estimate for the 95% confidence interval on \(c\) of [4.65, 7.38], so we can assert that, with very high probability, the convergence coefficient exceeds 4.0. Another way of interpreting this observation is that even if we use the upper estimate of the relative valence for ANAP, and the lower bound on \(\beta\), then the joint origin will still be a minimum of the vote share function for ANAP.

We now repeat the analysis for the election of 1999.
2.2 The 1999 Election

The empirical model presented in Table 4 estimated the electoral variance on the first axis (religion) to be 1.20 while on the second axis (nationalism) the electoral variance, $\sigma^2$, was 1.14, giving a total electoral variance, $\sigma^2$, of 2.34, with the covariance between the two axes equal to +0.78.

The electoral covariance matrix is the 2 by 2 matrix

$$\nabla_0 = \begin{bmatrix} 1.20 & 0.78 \\ 0.78 & 1.14 \end{bmatrix}. $$

For the model, the coefficient was 0.375, while the party with the lowest valence was FP with $FP = 0.16$. The vector of valences is:

$$(\Lambda_{FP}, \Lambda_{MHP}, \Lambda_{DYP}, \Lambda_{HAD), \Lambda_{ANAP}, \Lambda_{CHP}, \Lambda_{DSP}) $$

$$= (-0.16, +0.66, 0.0, -0.071, +0.34, +0.73, +0.72).$$

When all parties are located at the origin, the probability, $\rho_{FP}$, that a voter chooses FP under $M(\Lambda, \beta)$ is equal to

$$\frac{1}{1 + \exp(0.82) + \exp(0.16) + \exp(0.09) + \exp(0.5) + \exp(0.89) + \exp(0.88)}$$

$$= \frac{1}{11.27} = 0.08. $$

The standard error on $\Lambda_{FP}$ is 0.175, so the 95% confidence interval can be estimated to be [0.01, 0.15]. The FP vote share in 1999 was 15.41%, suggesting that the pure spatial model should be extended to include sociodemographic valences.

Now $2\beta(1 - 2\rho_{FP}) = 2\beta \times (1 - 2 \times (0.08)) = 2 \times 0.38 \times 0.84 = 0.64$, so the characteristic matrix of the FP is

$$C_{FP} = (0.64) \begin{bmatrix} 1.20 & 0.78 \\ 0.78 & 1.14 \end{bmatrix} - I$$

$$= \begin{bmatrix} -0.24 & 0.448 \\ 0.448 & -0.27 \end{bmatrix}.$$ 

and $c = 0.64 \times 2.34 = 1.49.$

Although $c < 2.0$, we can compute the eigenvalues of $C_{FP}$ to be $-0.74$ with minor eigenvector $(+1, -1.16)$ and $+0.23$, with major eigenvector $(+1, +0.896)$, giving a saddlepoint for the FP Hessian at the joint origin. As with the 2002 election, on the basis of the pure spatial model, we again expect all parties to align along the major eigenvector, at approximately 45 degrees to the religion axis. Note, however, that the standard error on $c$ is of order 0.22, so unlike the result for the election of 2002, we cannot assert that there is a high probability that the convergence coefficient exceeds 2. However, there is a probability exceeding 0.95 that one of the eigenvalues is positive.

In comparing the pure spatial models of the elections of 1999 and 2002, we note there is very little difference between the model predictions.
2.3 Extension of the model for Turkey

We now use the empirical joint model, \( M(\Lambda, \theta, \mu, \beta) \), in order to better model party positioning and to estimate the influence of party activists in a theoretical model which we denote \( M(\Lambda, \mu, \beta) \). The activist functions \( \mu = \{\mu_j : j \in P\} \) are presumed to be functions of party position, rather than exogenous constants. Theorem 1 of the Appendix shows that the first order condition for a local equilibrium, \( z^* = (z^*_1, \ldots, z^*_p) \), in the activist model, \( M(\Lambda, \mu, \beta) \), is given by the set of gradient balance conditions as specified in (6):

\[
\frac{dE_j^e}{dz_j}(z^*_j) + \frac{1}{2\beta} \frac{d\mu_j}{dz_j}(z^*_j) = 0.
\]  

Each term, \( \frac{d\mu_j}{dz_j}(z_j) \) is the marginal activist pull (or gradient) at \( z_j \), giving the marginal activist effects on party \( j \), while the gradient term \( \frac{dE_j^e}{dz_j}(z_j) = [z_j^{el} - z_j] \) is the gradient electoral pull on the party, at \( z_j \), pointing towards its weighted electoral mean, \( z_j^{el} \), as defined for party \( j \) in (5) in the Appendix:

\[
z_j^{el} = \sum_{i=1}^n \alpha_{ij} x_i, \text{ where } [\alpha_{ij}] = \left[ \frac{\rho_{ij} - \rho_{kij}^2}{\sum_{k \in N} (\rho_{kj} - \rho_{kij}^2)} \right].
\]  

The weighted electoral mean essentially weights voter policy preferences by the degree to which the sociodemographic valences influence the choice of the voter. The joint model, \( M(\Lambda, \theta, \beta) \), allows us to draw some inferences about equilibrium positions. First we note that in the joint model, the sociodemographic valences are substitutes for the relative valences. Table 7 shows that the only relative valence that is significantly non zero in 2002 is \( \Lambda_{AKP} \). A number of the sociodemographic valences are, however, very significant.

For example, Tables 6 and 7 show that the sociodemographic valences for HADEP (or DEHAP) by Kurdish voters were very high:

\[
\begin{align*}
\theta_{HADEP} \cdot \eta_{Kurd} &= 5.9 \text{ in } 1999 \\
\theta_{DEHAP} \cdot \eta_{Kurd} &= 6.0 \text{ in } 2002.
\end{align*}
\]

Keeping the other variables at their means in 2002, then changing \( \eta_{Kurd} \) from non-Kurd to Kurd increases the probability of voting for DEHAP from 0.013 to 0.45. The high significance level of the sociodemographic variables indicates that the joint electoral model would predict that DEHAP would move close to Kurdish voters who tend to be located on the left of the religion axis, and are also anti-nationalistic. The position marked HADEP in Figure 5 is consistent with this inference.

The joint model also shows that Alevi voters have very high sociodemographic valences for the CHP, with

\[
\begin{align*}
\theta_{CHP} \cdot \eta_{Alevi} &= 3.1 \text{ in } 1999 \\
\theta_{CHP} \cdot \eta_{Alevi} &= 2.6 \text{ in } 2002.
\end{align*}
\]
The Alevi are a non-Sunni religious community, who are adherents of Shia Islam rather than Sunni, and may be viewed as supporters of “Kemalism” or the secular state. Again, with other variables at their means, changing $Alevi$ from non-Alevi to Alevi increases the probability of voting for CHP in 2002 from $0.16$ to $0.63$. Thus the joint model indicates that the CHP will move to a vote maximizing position, on the left of the religious axis, again as indicated in Figure 5.

In the model $M(\Lambda, \theta, \beta)$, all the activist terms $\{\frac{d\mu_k}{dz_j}\}$ are zero, and we can infer from (1) that the first order balance condition reduces to

$$\frac{d\varepsilon_j}{dz_j} \equiv [z_j^e - z_j] = 0,$$

for each $j$.

Thus an LNE of the model $M(\Lambda, \theta, \beta)$ gives an estimate of $z^e = (z_1^e, \ldots, z_p^e)$. Note in particular that the probabilities $\{\rho_k(z_0)\}$ in (2) depend on voter $k$, so $z_0 \neq z^e$. Moreover, there may be many balance solutions for $M(\Lambda, \theta, \beta)$. Figure 5 gives one such LNE, obtained by simulation of the joint model. This equilibrium vector is given by:

$$z^e = \begin{bmatrix}
\text{Party} & CHP & MHP & DYP & HADEP & ANAP & AKP \\
x & -0.5 & 0.2 & 0.1 & -0.7 & -0.1 & 0.4 \\
y & -0.5 & 0.2 & 0.1 & -0.7 & -0.1 & 0.4 \\
\end{bmatrix}.$$ 

Notice that the party positions in this joint LNE are much closer to the estimated positions of the parties than the pure spatial LNE presented in Figure 4. Note that they also lie on the principal component given by an eigenvector $(1.0, 1.0)$. This is almost identical to the eigenvector for the LNE obtained for the pure spatial model. The estimated positions of the parties in Figure 3 are:

$$z^\ast = \begin{bmatrix}
\text{Party} & CHP & MHP & DYP & HADEP & ANAP & AKP \\
x & -2.0 & 0.0 & 0.0 & -2.0 & -0.2 & 1.0 \\
y & +0.1 & 1.5 & 0.5 & -1.5 & -0.1 & 0.1 \\
\end{bmatrix}.$$ 

Assuming that this vector is an LNE with respect to the full model, $M(\Lambda, \mu, \beta)$ involving activists, then by (13) in the Appendix, we can make the identification:

$$\frac{1}{2\beta} \begin{bmatrix}
\frac{d\mu_1}{dz_1}, \ldots, \frac{d\mu_p}{dz_p}
\end{bmatrix} = z^\ast - z^e = \begin{bmatrix}
\text{Party} & CHP & MHP & DYP & HADEP & ANAP & AKP \\
x & -1.5 & -0.2 & -0.1 & -1.3 & -0.1 & +0.6 \\
y & +1.5 & +1.3 & +0.4 & -0.8 & -0.1 & -0.3 \\
\end{bmatrix}.$$ 

Here, $\{\frac{d\mu_1}{dz_1}, \ldots, \frac{d\mu_p}{dz_p}\}$ are the marginal activist pulls at the equilibrium vector $z^\ast$. Under the hypothesis that the joint model with activists is valid, then
the difference between these two vectors gives us an estimate of the vector of marginal pulls on the parties:

The estimated activist pull on HADEP is very high, pulling the party to the left on the religion axis, and in an anti-nationalist direction on the y axis. Similarly, the estimated activist pull on the CHP is even higher, and we can infer that this is due to the influence of Alevi voters.

As a consequence, this asymmetry will cause Alevi activists to provide further differential support for the CHP. It is thus plausible that secular voters (on the left of the religious axis in Figures 1 and 3) would offer further support to the CHP, located close to them. This would affect the party’s marginal activist pull, and induce the CHP leader to move, in equilibrium, even further left.

For the AKP its relative valence \( \Lambda_{AKP} = 1.97 \), for the joint model is large and significant, so the weighted electoral mean, \( z''_{AKP} \), lies on the principal electoral axis, relatively close to the electoral center, as illustrated in Figure 5. We suggest that activist support for the AKP would move it slightly to the right on the religion axis, as well as in an anti-nationalism direction, as indicated in Figure 3.

In contrast, we might conjecture that the military provides activist support for the MHP on the nationalism axis, and this will move the party to the left in a secular direction, and north on the nationalism axis, as in Figure 3.

Overall, we note that we can expect activist valence to strongly influence party positioning, and we can proxy this support to some degree using the sociodemographic variables. Notice that the sociodemographic variables are estimated at the vector \( z^* \), so the estimated sociodemographic valences have been influenced by activist support. The different LNE obtained from the joint model are therefore merely hypothetical solutions to the vote maximizing game involving the parties, based on some empirical assumptions about the underlying nature of the important sociodemographic groups in the polity.

2.4 General remarks on Turkish elections

Although we have not performed a MNL analysis of the 2007 election, it seems obvious that some of the changes in the nature of party strategies were due to changes in the electoral laws. The election results of 1999 were based on an electoral system that was quite proportional, whereas in 2002 and 2007, the electoral system was highly majoritarian. In 2002, for example, the AKP gained 66% of the seats with only 34% of the vote, while in 2007 it took 46.6% of the vote and 340 seats (or 61.8%), reflecting the continuing high valence of Erdogan. Similarly, the CHP went from about 9% of the vote in 1999 (and no seats) to 19% of the vote in 2002, and 32% of the seats. This is mirrored by the increase in the valence estimates of the joint model from \( \Lambda_{CHP} = -0.673 \) in 1999 to \( \Lambda_{CHP} = 1.103 \), in 2002. In contrast the MHP went from 18% of the vote in 1999 to 8% in 2002, while \( \Lambda_{MHP} \) for the joint model fell from 2.5 to 1.7. The turn around in the vote share of the MHP between 2002 and 2007 could be a result of increasing support for this party from nationalist activist groups in an attempt to offset the high valence and electoral support for the AKP in
2002. Indeed, the increased concentration of the vote share between 1999 and 2007 may be a consequence of the greater significance of activist influence as the electoral system became more majoritarian.\textsuperscript{26} One strategic consequence of the change in the electoral law was that in the 2007 election, the Kurdish Party (now called the Freedom and Solidarity Party, DTP) avoided the 10 percent cut-off by contesting the election as independents, and were thus able to win 24 seats.

In such a non-proportional electoral system there are incentives for members of different sociodemographic groups to to engage in strategic voting. There is some indication from the formal model that the intensity of the political contest between secularist, nationalist and religious activist groups had increased prior to 2007, and recent events suggest that this is continuing.

After the 2007 election, Abdullah Gul, Erdogan's ally in the AKP was elected as the country's 11th president, despite strong opposition from the army and many secular interests. In late February, 2008, the Turkish military invaded the Kurdish controlled territory in north west Iraq in an attempt to destroy the bases of the P.K.K. (the Kurdistan Workers' Party). The secular Constitutional Court has also considered banning many members of the AKP. In September 2008, Turkey formed a Caucasus Stability and Cooperation Platform with five neighboring countries, in response to Russian aggression in Georgia, and President Gul visited Armenia, one of the countries in the Platform. On January 30, 2009, Erdogan returned home from the World Economic Forum in Davos after walking out of a televised debate with Shimon Peres, the Israeli president, over Israel's war on the Gaza Strip. The moderator had refused to allow Erdogan to rebut Peres' justification of the war. Erdogan was welcomed back in Turkey as a hero.

However, more secular voters have begun to worry that Erdogan had become more autocratic, and in the municipal elections in March, 2009, the vote for the AKP dropped from 47% to 39%. It appeared that the Turkish electorate had divided geographically into four different political regions: a liberal, secular litoral, a conservative interior, with a nationalistic center, and a Kurdish nationalist southeast.\textsuperscript{27}

In his visit to Turkey in April 2009, Barack Obama made it clear that in his view, Turkey should become a member of the European Union. At the same time, he urged Turkey to undertake more democratic reforms. Although Turkey has many of the characteristics of a full democracy, it does appear to be subject to the pressure of activist groups, such as the military.

\section{Concluding Remarks}

Recent works by Acemoglu and Robinson (2006), Boix (2003), and Przeworski et al. (2000) have explored the transition from autocratic regimes to democracy.

\textsuperscript{26}The Herfindahl concentration measure of the vote shares went from 0.11 in 1999 to 0.16 in 2002 to 0.27 in 2007.

\textsuperscript{27}Asli Aydintasbas in the \textit{New York Times}, April 7, 2009.
A recent contribution by Epstein et al (2006) has emphasized the existence of the category of “partial democracies.” These exhibit mixed characteristics of both democratic and autocratic regimes. In fact, Epstein et al. give Turkey as a prime illustration of the possible degree of democratic volatility of a regime. They observe that, in terms of Polity IV scores, Turkey fell from being a full democracy to an autocracy first in the mid 1960’s and again in the early 1980’s, and since then has hovered between partial and full democracy. Epstein et al (2006:564) also comment, on the basis of their empirical analysis, that “the determinants of the behavior of partial democracies elude our understanding.”

Models of democratic transitions have tended to consider a single economic axis, and to utilize the notion of a median citizen as the unique pivotal player. While these models have been illuminating, we believe it necessary to consider policy spaces of much higher dimension and to utilize a stochastic model so as to emphasize the aspect of uncertainty.

The analysis of Turkey in this paper indicates that both religion and nationalism define the political space. Obviously enough the military in Turkey can be represented by a pro-nationalist, pro-secular position, far from the AKP, and it is this phenomenon which means that Turkish politics cannot be understood in terms of a median voter. Modeling partial democracies would seem to require a very explicit analysis of the power of activist groups. This paper has applied a theoretical stochastic model to present an empirical analysis of elections in Turkey. The main theoretical point that emerges is that there is no evidence of a centripetal tendency towards an electoral center. It is consistent with this analysis that activist groups will tend to pull the parties away from the center. The joint model with sociodemographic valence has been used to suggest how it is possible to estimate the influence of activists pulling the parties towards policy positions preferred by the activists.

4 Appendix: Formal and Empirical Electoral Models

4.1 The Model with Activists

The electoral model presented here is an extension of the multiparty stochastic model of McKelvey and Patty (2006), modified by inducing asymmetries in terms of valence. The justification for developing the model in this way is the empirical evidence that valence is a natural way to model the judgements made by voters of party leaders and candidates. There are a number of possible choices for the appropriate model for multiparty competition. The simplest one, which we first present, is that the utility function for leader \( j \) is proportional to the popular support, \( V_j \), of the party in the election.\(^{29}\) With this assumption, we

---

\(^{28}\)Schofield and Sened (2006) found the electoral model for Israel to be very similar to Turkey, with two electoral axes, religion and security.

\(^{29}\)The popular support may be identical to the vote share in a democratic election, or may be weighted by individual characteristics, such as domicile, income or ownership of land, in
can examine the conditions on the parameters of the stochastic model which are necessary for the existence of a pure strategy Nash equilibrium (PNE). Because the vote share functions are differentiable, we use calculus techniques to obtain conditions for positions to be locally optimal. Thus we examine what we call local pure strategy Nash equilibria (LNE). From the definitions of these equilibria it follows that a PNE must be a LNE, but not conversely. A necessary condition for an LNE is thus a necessary condition for a PNE. A sufficient condition for an LNE is not a sufficient condition for PNE. Indeed, additional conditions of concavity or quasi-concavity are required to guarantee existence of PNE.

The stochastic model essentially assumes that candidates cannot predict vote response precisely, but that they can estimate the effect of policy proposals on the expected vote share. In the model with valence, the stochastic element is associated with the weight given by each voter, \( i \), to the average perceived quality or valence of each candidate. We also consider a formal model where the perceptions of the leader qualities vary across different sociodemographic groups in the society.

The data of the spatial model is a distribution, \( \{x_i \in X\}_{i \in N} \), of voter ideal points for the members of the electorate, \( N \), of size \( n \). We assume that \( X \) is a subset of Euclidean space, of dimension \( w \) with \( w \) finite. Without loss of generality, we adopt coordinate axes so that \( \frac{1}{n} \sum x_i = 0 \). By assumption \( 0 \in X \), and this point is termed the electoral mean, or alternatively, the electoral origin. Each of the parties in the set \( P = \{1, \ldots, j, \ldots, p\} \) chooses a policy, \( z_j \in X \), to declare prior to the specific election to be modeled. Let \( z = (z_1, \ldots, z_p) \in X^p \) be a typical vector of party policy positions.

Given \( z \), each citizen, \( i \), is described by a utility vector

\[
\mathbf{u}_i(x_i, z) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))
\]

where

\[
u_{ij}(x_i, z_j) = \lambda_j + \mu_j(z_j) - \beta||x_i - z_j||^2 + \varepsilon_j = u_{ij}^*(x_i, z_j) + \varepsilon_j. \tag{3}
\]

Here \( u_{ij}^*(x_i, z_j) \) is the observable component of utility. The constant term, \( \lambda_j \), is the fixed or intrinsic valence of party \( j \). The function \( \mu_j(z_j) \) is the component of valence generated by activist contributions to agent \( j \). The term \( \beta \) is a positive constant, called the spatial parameter, giving the importance of policy difference defined in terms of a metric induced from the Euclidean norm, \( ||\cdot|| \), on \( X \). The vector \( \varepsilon = (\varepsilon_1, \ldots, \varepsilon_j, \ldots, \varepsilon_p) \) is the stochastic error, whose multivariate cumulative distribution will be denoted by \( \Psi \). The notation \( \lambda_j + \mu_j(z_j) \) is intended to imply that this is the average valence for party \( j \) among the electorate, but the realized valence is a distributed by \( \Psi \). The most common assumption in empirical analyses is that \( \Psi \) is the Type I extreme value distribution (sometimes called Gumbel). This cumulative distribution has the closed form

\[
\Psi(x) = \exp \left\{ -\exp \left[ -x \right] \right\}.
\]

non-democratic polities.
The theorems presented in this appendix are based on this assumption. This distribution assumption is the basis for much empirical work based on multinomial logit estimation (Dow and Endersby, 2004). It is assumed that the intrinsic valences are all finite, and the valence vector

\[ \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p) \] satisfies \( \lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1 \).

Voter behavior is modeled by a probability vector. The probability that a voter \( i \) chooses party \( j \) at the vector \( z \) is

\[ \rho_{ij}(z) = \Pr\{u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)\}, \text{ for all } l \neq j \].

Here \( \Pr \) stands for the probability operator generated by the distribution assumption on \( \varepsilon \).

With this distribution assumption on \( \varepsilon \), it follows, for each voter \( i \); and leader \( j \), that

\[ \rho_{ij}(z) = \exp\left[u_{ij}(x_i, z_j) - \sum_{k=1}^{p} \exp u_{ik}(x_i, z_k)\right]. \] (4)

For any voting model the likelihood of a model is

\[ L = \prod_{i \in N, j \in P} \rho_{ij}(z), \]

while the log likelihood of the model is \( \log_e(L) \). Clearly as \( L \) approaches 0 then \( \log_e(L) \) approaches \(-\infty\).

To compare two models, \( M_1 \) and \( M_2 \), the Bayes Factor is \( L(M_1)/L(M_2) \) and the log Bayes factor of \( M_1 \) against \( M_2 \) is \( \log_e(L(M_1)) - \log_e(L(M_2)) \). A log Bayes factor over 5.0 for \( M_1 \) against \( M_2 \) is considered strong support for \( M_1 \) (Kass and Raftery, 1995).

The expected popular support for leader \( j \) is

\[ V_j(z) \equiv \sum_{i \in N} s_{ij} \rho_{ij}(z). \]

Here \( \{s_{ij}\} \) are different weights that can be associated with different voters. In the case all weights are equal to \( \frac{1}{n} \), we call the model egalitarian. For development of the model we now regard \( V = \{V_j : j \in P\} \) as a set of vote share functions, and identify \( V \) as a differentiable profile function, \( V : X^p \to \mathbb{R}^p \). We denote the egalitarian profile function as \( V_e \).

In this stochastic electoral model it is assumed that each party \( j \) chooses \( z_j \) to maximize \( V_j \), conditional on \( z_{-j} = (z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p) \).

Thus a vector \( \mathbf{z}^* = (z_1^*, \ldots, z_{j-1}^*, z_j^*, z_{j+1}^*, \ldots, z_p^*) \) is called a local strict Nash equilibrium (LSNE) if each \( z_j \) strictly locally maximizes \( V_j \), conditional on \( \mathbf{z}_{-j} \).
while \( z^* \) is a local weak Nash equilibrium (LNE) if each \( z_j \) weakly locally maximizes \( V_j \), conditional on \( z_{-j} \). The notion of LSNE is convenient so as to avoid degeneracy problems associated with the Hessians.

In the same way the vector \( z^* \) is a strict (or weak) pure strategy Nash equilibrium (PSNE or PNE) if each party \( j \) chooses \( z_j \) to strictly (or weakly) maximize \( V_j \) on \( X \).

Now assume that the vector \( z \) is fixed, and let \( \rho_{ij}(z) = \rho_{ij} \) be the probability that \( i \) picks \( j \). Define the \( p \) by \( n \) matrix array of weights by

\[
[\alpha_{ij}] = \left[ \frac{s_{ij}[\rho_{ij} - \rho_{ij}^2]}{\sum_{k \in N} s_{kj}[\rho_{kj} - \rho_{kj}^2]} \right] \tag{5}
\]

The vector \( \sum_i \alpha_{ij} x_i \) is a convex combination of the set of voter ideal points and is called the weighted electoral mean for party \( j \). Define

\[
z_{elj} \equiv \sum_{i=1}^{n} \alpha_{ij} x_i \text{ and } \frac{dE_j}{dz_j}(z_j) \equiv \left[ z_{elj} - z_j \right].
\]

Then the balance equation for \( z_j^* \) is given by the expression

\[
\frac{dE_j}{dz_j}(z_j^*) + \frac{1}{2\beta} \frac{d\mu_j}{dz_j}(z_j^*) = 0. \tag{6}
\]

The term \( \frac{dE_j}{dz_j}(z_j) \) is the marginal electoral pull of party \( j \) at the point \( z_j \) and can be regarded as a gradient vector, at \( z_j \), pointing towards the weighted electoral mean of the party. (Note that this electoral pull depends on the positions of all leaders.) When \( z_j \) is equal to the weighted electoral mean then the electoral pull is zero. The gradient vector \( \frac{d\mu_j}{dz_j}(z_j) \) is called the marginal activist pull for party \( j \) at \( z_j \).

If \( z^* = (z_1^*, \ldots, z_j^*, \ldots, z_p^*) \) is such that each \( z_j^* \) satisfies the balance equation then call \( z^* \) a balance solution. The balance solution requires that the electoral and activist gradients are directly opposed, for every party leader.

The model just presented is denoted \( M(\lambda, \mu, \beta; \Psi; V) \). Schofield (2006a) proves the following theorem for this model.

**Theorem 1.**

Consider the electoral model \( M(\lambda, \mu, \beta; \Psi; V) \) based on the distribution, \( \Psi \), including both intrinsic and activist valences, and defined by the family \( V \) of vote share functions.

(i) The first order condition for \( z^* \) to be an LSNE is that it is a balance solution.

(ii) If all activist valence functions are sufficiently concave\(^{30}\), then a balance solution will be a PNE.

\(^{30}\)By this we mean that the eigenvalues of the activist functions are negative and of sufficient magnitude everywhere.
In the full activist model, $M(\lambda; \mu; \beta; \Psi; V)$, with valence functions $\{\mu_j\}$ that are not identically zero or constant, then it is the case that generically $z_0$ cannot satisfy the first order conditions for LNE even when $V$ is egalitarian. Instead the vector $\frac{d\mu_j}{dz_j}$ “points towards” the position at which the activist valence for leader $j$ is maximized. When this marginal or gradient vector, $\frac{d\mu_j}{dz_j}$, is increased (as activist groups become more willing to contribute to leader $j$) then the equilibrium position is pulled away from the weighted electoral mean of the leader, and we can say the “activist effect” for the leader is increased. In the case of two opposed leaders, $j$ and $k$, if the activist valence functions are fixed, but the intrinsic valence, $\lambda_j$, is increased, or $\lambda_k$, is decreased, then the weighted electoral mean, $z_{el}^j$, approaches the electoral origin. Thus the local equilibrium of leader $j$ is pulled towards the electoral origin. We can say the “electoral effect” is increased.

4.2 The Egalitarian Model without Activists

In the case that the activist valence functions are identically zero, or constant, we denote the model by $M(\lambda; \beta; \Psi; V)$. The key consideration for the egalitarian model, $M(\lambda; \beta; \Psi; V)$, when all voter weights are identical, is whether the electoral origin is a LSNE. For this model it can be shown that if all parties are at the same position, so $z^* = (z^*_1, z^*_2, ..., z^*_n)$ then every $\rho_{ij}(z^*) : i \in N$ is independent of $i$, and can thus be written $\rho_j(z^*)$. This implies that all $\alpha_{ij}$ in (5) are identical at $z^*$ and equal to $1/n$. Thus, when there is only intrinsic valence, the equation $z_j^* = \frac{1}{n} \sum x_i$ satisfies the balance solution for all $j$. By an appropriate coordinate change, we can assume $\frac{1}{n} \sum x_i = 0$. In this case, all marginal electoral pulls are zero at $z_0 = (0, ..., 0)$, so $z_0$ satisfies the first order conditions. However, to determine whether $z_0$ is an LNE it is necessary to examine the Hessians of the vote share functions.

We first define the electoral covariance matrix, $\nabla_0$, and then use $\nabla_0$ to define the convergence coefficient of the model $M(\lambda; \beta; \Psi; V_e)$. Let $X = \mathbb{R}^w$ be endowed with a system of coordinate axes $r = 1, ..., w$. For each coordinate axis let $\xi_r = (x_{1r}, x_{2r}, ..., x_{nr})$ be the vector of the $r^{th}$ coordinates of the set of $n$ voter ideal points. The scalar product of $\xi_r$ and $\xi_s$ is denoted $(\xi_r, \xi_s)$. Let $(\sigma_r \cdot \sigma_s) = \frac{1}{n} (\xi_r, \xi_s)$ be the electoral covariance between the $r^{th}$ and $s^{th}$ axes, and $\sigma_s^2$ be the variance on the $s^{th}$ axis.

(i) The symmetric $w \times w$ electoral covariance matrix about the origin is denoted $\nabla_0$ and is defined by

$$\nabla_0 \equiv \left[ (\sigma_r \cdot \sigma_s) \right]_{r=1,...,w}^{s=1,...,w}.$$

(ii) The total electoral variance is $\sigma^2 \equiv \sum_{s=1}^w \sigma_s^2 = \text{trace}(\nabla_0)$.

(iii) At the vector $z_0 = (0, ..., 0)$ the probability $\rho_{ij}(z_0)$ that $i$ votes for party
\( j \) is independent of \( i \), and is given by

\[
\rho_j = \left[ 1 + \sum_{k \neq j} \exp[\lambda_k - \lambda_j] \right]^{-1}. \tag{7}
\]

(iv) The Hessian of the egalitarian vote share function of party \( j \) at \( z_0 \) is a positive multiple of the \( w \) by \( w \) characteristic matrix,

\[
C_j \equiv 2\beta(1 - 2\rho_j)\nabla_0 - I. \tag{8}
\]

(Here \( I \) is the identity matrix.)

The convergence coefficient of the egalitarian model, \( \mathcal{M}(\lambda; \beta; \Psi; V_e) \), is defined to be

\[
c \equiv c(\lambda; \beta; \Psi; V_e) \equiv 2\beta[1 - 2\rho_j]\sigma^2. \tag{9}
\]

**Theorem 2.**

Consider the electoral model \( \mathcal{M}(\lambda; \beta; \Psi; V_e) \) where all activist valence functions are zero (or constant) and \( V_e \) is the egalitarian party profile.

(i) The joint origin \( z_0 = (0, \ldots, 0) \) satisfies the first order condition to be a LSNE for this model.

(ii) In the case that \( X \) is \( w \) dimensional then the necessary condition for \( z_0 \) to be a LNE for this model is that \( c(\lambda; \beta; \Psi; V_e) \leq w \).

(iii) In the case that \( X \) is 2 dimensional, a sufficient condition for \( z_0 \) to be a LSNE for this model is that \( c(\lambda; \beta; \Psi; V_e) < 1 \).

The proof and some applications of Theorem 2 are given in Schofield (2007a,b)

### 4.3 Empirical Models

In empirical models with intrinsic valence alone it is necessary to estimate the model with respect to the valence of a baseline party, say \( k^* \). We set \( \Lambda_j = \lambda_j - \lambda_{k^*} \), and call these the relative valences. We denote this egalitarian model by \( \mathcal{M}(\Lambda, \theta; \Psi; V_e) \).

At the joint origin \( z_0 \), we see that

\[
\rho_{ij}(z_0) = \frac{\exp(\lambda_j)}{\sum_{k=1}^{p} \exp(\lambda_k)} = \frac{\exp(\lambda_j - \lambda_{k^*})}{\sum_{k=1}^{p} \exp(\lambda_k - \lambda_{k^*})} = \frac{\exp(\Lambda_j)}{\sum_{k=1}^{p} \exp(\Lambda_k)} \tag{10}
\]

is again independent of the individual, \( i \), and can be written as \( \rho_j \).

To estimate the standard error on \( \rho_j \), we use Taylor’s Theorem, which asserts that

\[
\rho_j(\Lambda_j + h) = \rho_j(\Lambda_j) + h \frac{d\rho_j}{d\Lambda_j} = \rho_j(\Lambda_j) + h\rho_j(1 - \rho_j). \tag{11}
\]
4.4 Empirical Models with Sociodemographic Valences

As described in the body of the paper, in empirical applications with sociodemographic variables, we typically assume that $V$ is the egalitarian party profile function, $V_e$, so the model $M(\Lambda, \theta, \beta; \Psi; V_e)$ is based on the assumption that voter utility has the form

$$u_{ij}(x_i, z_j) = \Lambda_j + (\theta_j \cdot \eta_i) - \beta \|x_i - z_j\|^2 + \varepsilon_j.$$

From (5), an immediate consequence of this, however, is that even when all parties are at the origin, then $\rho_{kj}(z_0) : k \in N$ will depend on $k$. We can infer that, generically, $z_0$ will not satisfy the first order condition for LNE. However, the joint empirical model, $M(\Lambda, \theta, \beta; \Psi; V_e)$, assumes that the sociodemographic effects are independent of party positions, and this implies $\frac{d\mu_j}{dz_j} = 0$, for all $j$. Using (6), we infer that the various LNE obtained by simulation of the joint model constitute a set of vectors of weighted electoral means: $\{z^{el^1, ..., z^{el^p}}\}$.

Assuming that the estimated party positions are given by the vector $z^* = (z_1^*, ..., z_p^*)$ and that this is in equilibrium with respect to the full activist model, then choosing one joint LNE, $z^{el}$, gives an estimate of

$$\left[ z_j^{el} - z_j^* \right] = \frac{d\mathcal{E}_j^*}{dz_j} (z_j^*) = -\frac{1}{2\beta} \left[ \frac{d\mu_j}{dz_j} \right].$$

Thus

$$\left[ z^* - z^{el} \right] = \frac{1}{2\beta} \left[ \frac{d\mu_1}{dz_1}, ..., \frac{d\mu_p}{dz_p} \right].$$

(12)

(13)

This observation suggests how the gradients of the activist valence functions may be inferred from a comparison of LNE of the joint empirical model with the estimated political configuration.
5 References


Hinich M (1977) Equilibrium in spatial voting: the median voter result is an artifact. J. Econ Theory 16:208-219


Table 1  Turkish election results 1999

<table>
<thead>
<tr>
<th>Party Name</th>
<th>% Vote</th>
<th>Seats</th>
<th>% Seats</th>
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<tbody>
<tr>
<td>Democratic Left Party</td>
<td>DSP</td>
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<td>129</td>
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<tr>
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Total 550

Table 2  Turkish election results 2002

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Table 3  Turkish election results 2007

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<th>% Seats</th>
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\textsuperscript{31}Twenty-four of these “independents” were in fact members of the DTP— the Kurdish Freedom and Solidarity Party.
Table 4. Pure Spatial Model of the Turkish election 1999.

<table>
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<tr>
<th>Party Name</th>
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<td>0.147</td>
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<td>Virtue Party FP</td>
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<td>0.175</td>
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<td>0.336*</td>
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<td>True Path Party DYP</td>
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<td></td>
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<tr>
<td>Republican People’s Party CHP</td>
<td>0.734*</td>
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<td>People’s Democracy Party HADEP</td>
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</table>

(Normalized with respect to DYP)

Spatial coefficient $\beta$ 0.375* 0.088 4.26
Convergence coefficient $c$ 1.49 0.22 6.77

$n = 635$
Log likelihood = $-1183$

*=Significant with probability < 0.001.

Table 5. Pure Spatial Model of the Turkish election 2002

<table>
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<td>True Path Party DYP</td>
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(Normalized with respect to DYP)

Spatial coefficient $\beta$ 1.52* 0.12 12.66
Convergence coefficient $c$ 5.94* 0.27 22.0

$n = 483$
Log likelihood = $-737$

*=Significant with probability < 0.001.
Table 6  Joint Multinomial Logit Analysis of the 1999 Election in Turkey,  
(normalized with respect to DYP)  

<table>
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<th>Variable</th>
<th>Party</th>
<th>Coefficient</th>
<th>95% Confidence Interval</th>
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<td>Std Err</td>
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n=635  
*:prob<0.001  
Log marginal likelihood = -1178
Table 7. Multinomial Logit Analysis of the 2002 Election in Turkey
(normalized with respect to DYP)

<table>
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<th>Coefficient</th>
<th>95% Confidence Interval</th>
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*n=483  *: prob < 0.001  Log marginal likelihood = -732
Figure 1: Party positions and voter distribution in Turkey in 1999.

Figure 2: Party positions and voter distribution in Turkey in 2002.
Figure 3: A Local Nash Equilibrium for the pure spatial model in 2002

Figure 4: A Local Nash Equilibrium for the joint model in 2002