Structural cohesion is the appropriate concept for units of a network identified by (*k*-path) multiconnectivities and equivalent (*k*-cut) connectivities at each distinct level *k*. These two aspects give it an unusually strong and scale-free basis of network measurement. Scale free in this context means that the consequences of relational cohesion should be similar from the smallest to the largest networks, and can be, in principle, similarly measured.

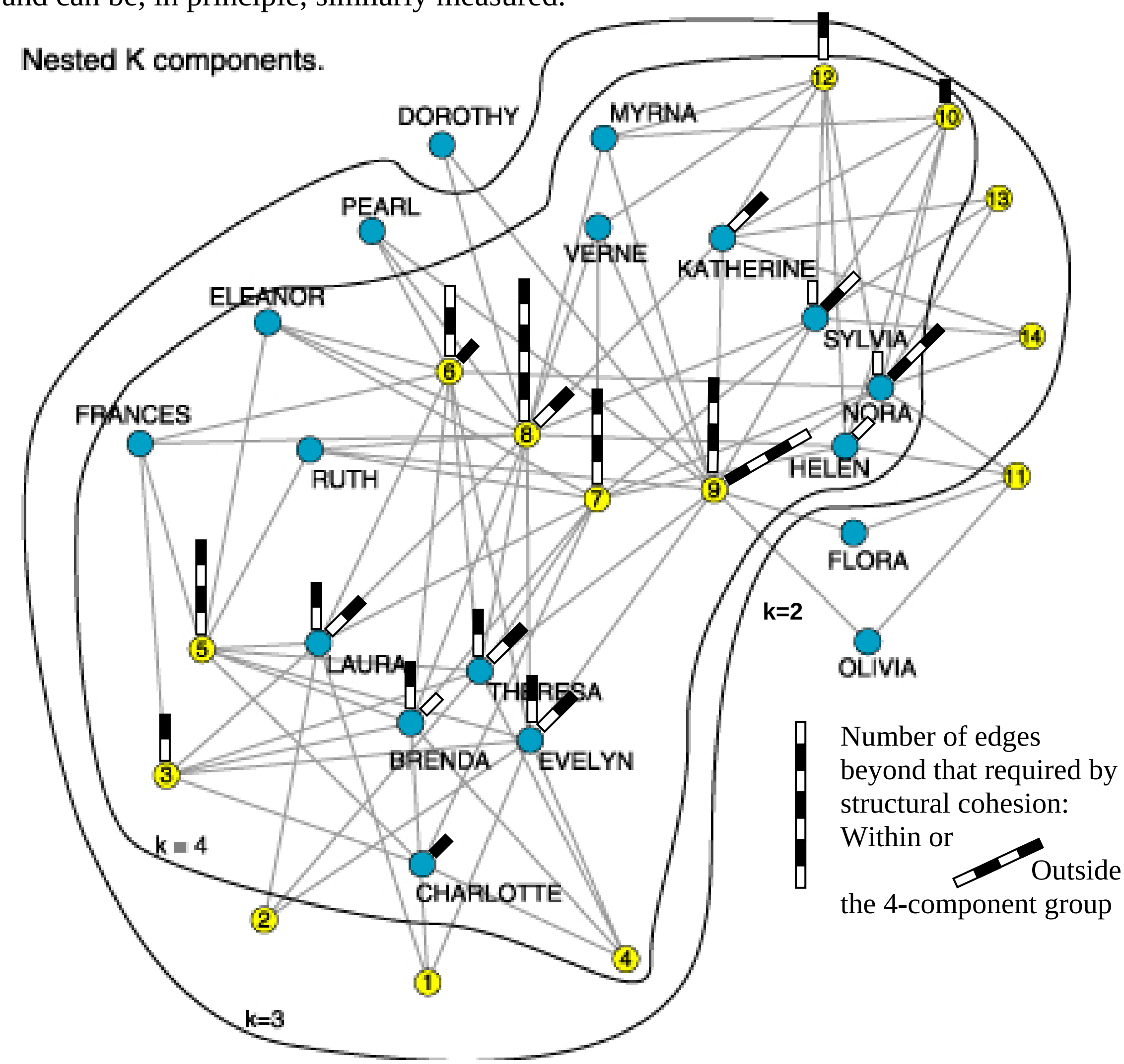


Fig. 1. Results of Applying Cohesive Blocking to the Southern Women Network; extra-edges within the appropriate cohesive group are shown by vertical yardsticks and extra edges outside the cohesive group by diagonal yardsticks

Consider figure 1, modified from White, Owen-Smith, Moody and Powell (2004), showing the co-attendance network of 18 women in one of the inventories of social events in the year 1936 in the southern town studied by Davis, Gardner and

Gardner (1941). Women are represented by the named nodes and events by nodes with numbers. Three *k*-components (maximal sets of nodes with different levels of multiconnectivity) are identified. Each node has a pair of yardsticks that show its number of edges *beyond* those required by its *k*-component. Vertical yardsticks count the extra edges *within* the 4-component, and diagonal show the extra edges *outside* the 4-component. The hubs of this network are the three central events, 7-9, with the highest yardsticks. Four women on the bottom left have 2 excess edges within the 4-component, enclosed in the contour lines labeled *k*=4, formed by 12 women and 9 events that are inseparable without removal of 4 or more of its nodes. Events 5 and 3 on the lower left have 4 and 2 excess edges within the 4-component, as does event 12 in the upper right. Event 9 and Nora have the most extra edges to nodes outside the 4-component.

A larger and less cohesive subset, labeled *k*=3 and enclosing the 4-component, consists of a unique 3-component. It has an additional woman (Pearl) and four additional events (1, 2, 13, and 14). Every pair of its nodes is connected by at least three node-independent paths (so it also cannot be disconnected by removal of 2 or fewer of its nodes). The entire network, with three additional women (Dorothy, Flora, and Olivia) and one extra event (11), constitutes a 2-component (not separable except by removal of *k*=2 or more nodes) that in this case is also the 1-component of the network.

Individuals may be more or less strongly embedded in structurally cohesive groups. Structurally cohesive groups may be embedded in one another hierarchically; differing by the minimum size of node cutsets needed to disconnect them. Within structurally cohesive groups, however, further identification of cohesive subgroups can be made.

Moody and White (2003:106, 111) define the *embeddedness* of subgroups in terms of the subgroups that result from minimum cutsets of nodes. The idea of the Moody-White cohesive blocking algorithm is given in their appendix (p. 123):

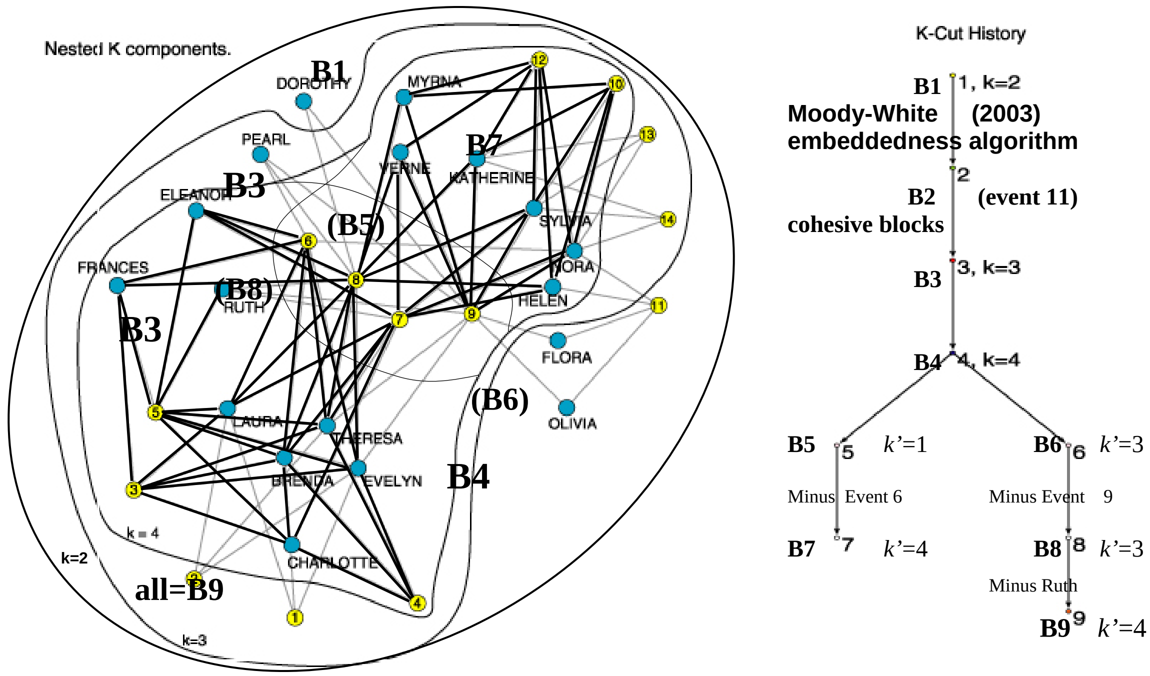
> We can identify cutsets in a network as follows:
> (1) Identify the connectivity, *k*, of the input graph.
> (2) Identify all *k*-cutsets at the current level of connectivity.
> (3) Generate new graph components based on the removal of these cutsets (nodes in the cutset belong to both sides of the induced cut).
> (4) If the graph is neither complete nor trivial, return to 1; else end.
>
> This procedure is repeated until all nested connectivity sets have been enumerated.

Moody and White (2003: 112) "identify clear dimensions of embeddedness that would admit to empirical operationalization" of concepts at the center of sociology (p. 112), citing Granovetter:

> Granovetter (1992) points to a key division between "local" and "structural" embeddedness:
>
> > "Embeddedness" refers to the fact that economic action and outcomes, like all social action and outcomes, are affected by actors' dyadic (pairwise) relations *and* by the structure of the overall network of relations. As a shorthand, I will refer to these as the relational and the structural aspects of embeddedness. The structural aspect is especially crucial to keep in mind because it is easy to slip into "dyadic atomization," a type of reductionism. (Granovetter 1992, P. 33, italics in original)
>
> Granovetter (1992) further specifies his understanding of structural embeddedness as the degree to which actors are involved in cohesive groups:
>
> > [T]o the extent that a dyad's mutual contacts are connected to one another, there is more efficient information spread about what members of the pair are doing, and thus better ability to shape behavior. Such cohesive groups are better not only at spreading information, but also at generating normative, symbolic, and cultural structures that affect our behavior." (P. 35)
>
> Granovetter's concept invokes transitivity (Davis 1963; Holland and Leinhardt 1971; Watts 1999), focusing on the pattern of relations among a focal actor's contacts. One need not limit structural embeddedness to an actor's direct neighborhood, however, but can extend the notion of embeddedness in a cohesive group to the wider social network (Frank and Yasumoto 1998). The concept of *k*-connected groups provides a clear operationalization of a structural aspect of embeddedness through the degree to which actors' partners (or their partners' partners) are connected to one another through multiple independent paths. As such, because cohesive groups are nested within one another, then each successive *k*-connected set is more deeply embedded within the network. This deep connectivity nicely captures the intuitive sense of being involved in relations that are, in direct contrast to "armslength" relations, structurally embedded in a social network (Uzzi 1996). As such, one aspect of structural embeddedness—the depth of involvement in a cohesive structure—is captured by this nesting.

Using their algorithm, Moody and White define "an actor's nestedness in a social network" – cohesive *embeddedness* – as "*the deepest cutset level within which the* *actor resides*" (P.112). Fig. 2 shows the same network as in figure 1, now with the full results of the Moody-White cohesive blocking algorithm (Moody 1999), including the results for the finer levels of structural embeddedness. In this example, only the removal of events 6-8 and Nora or events 6-9 splits the 4-component into two nontrivial groups (there are also 8 cuts of four nodes that separate only one woman or one event). Since only one 4-event removal splits the 4-component, the two subgroups that are split – **group 1** associated with blocks **B6**, **B8** and **B9** and **group 2** attached to blocks **B5** and **B7** – are more cohesively embedded than any others within the 4-component. The tree to the right of the figure shows successive cohesive splits that correspond, first, to levels of multiconnectivity (cohesive blocks **B1, B3, B4**), and second, to further structurally embedded subgroups (**B5-B9**). These latter separations are shown in the network of fig. 2 by the dotted contours within the 4-component. To illustrate how embedded subsets occur within the **B4** 4-component, after identifying events 6-9 as a cutset the separated nodes in **B5** for events 6-15, for example (recalling that the nodes in the cutset belong to both sides of the cut), the resulting block is only 1-connected. Removal of event 6 from **B5** results in a 4-component, **B7,** however, while the removal of event 9 from **B6** results in a 3-component, **B8**.

Two kinds of results can be extracted from cohesive blocking, as in the WOMP04 (**DGG41**) example: First, the assignment of people to groups, and second, rankings of core versus peripheral members of each group. These relationships among resulting blocks and women are shown in at the bottom of fig. 2. The relationships among the blocks themselves are quite simple, as is the separation into two groups on the basis of cohesive embedding. How the individuals in blocks 2 and 3 fit into the block structure, however, is complex. They include women who are peripheral to 4-component **B4**, who are not assigned to a particular group by structurally cohesive blocks alone. Pearl in block 3, however, attends events 6 (in **B6** but not **B7**), 8 (a member of all blocks), and 9 (in **B5** but not **B8**) and so has an overlapping membership in groups 1 and 2. Because these blocks have more participants from group 1 than group 2, Pearl can be said to lean towards group 1 rather than 2. Dorothy, Olivia and Flora, in this sense, lean toward group 2.



Key: Dark lines are those within the 4-connected blocks **B7** and **B9**. The k-components identified by the Moody-White (2003) algorithm within the 2-mode graph of events and actors are enclosed in cohesive contours labeled *k*=2, 3 and 4. The tree at right for *k*-cut history shows the order in which the algorithm finds first the k-components (again labeled *k*=2, 3 and 4) and then the remaining embedded subsets found after the 4-cut of block 4 (with both sides, starting with **B5** and **B6**, retaining the cutset). The dotted lines in the graph separate the 3- and 4-connected sets identified at steps 5 and 7 by the algorithm, and the 1-, 3-, and 4-connected sets identified at steps 6, 8 and 9. The only other four node 4-component split (also found by the algorithm but not shown) places temporal event 6 in the lower-left embedded set and removes Nora from the upper-right embedded set. The connectivities *k'*=1, 3, 4 for **B5-B9** in the K-Cut History reflect the fact the once the **B4** 4-component is split up, the immediately resulting blocks are of lower connectivity, but as they are split *k*-connectedness increases.

2-component **Block 1** (inclusive of those below)
16. Dorothy 17. Olivia 18. Flora
3-component **Block 3** (inclusive of those below)
8. Pearl
4-component **Block 4** (inclusive of those below)

Group 1
Core ⟶ Periphery ⟶

Group 2
⟵ Periphery ⟵ Core

Block 9 **Block 8** **Block 6**
1. Evelyn 9. Ruth 8.
2. Laura plus B9 16.

Block 5 **Block 7**
plus B7 10. Verne
11. Myra*

3. Theresa
4. Brenda
5. Charlotte
6. Francis
7. Eleanor

17.
18.

12. Katherine
13. Sylvia
14. Nora
15. Helen
*(misspelled in fig.)

Fig. 2 Additional Cohesive Embedding in the Southern Women Data: Groups, Core-Periphery Continua, and Hierarchical Inclusions of Cohesive Blocks

Structurally cohesive groups are not mutually exclusive of one another, but are embedded in a hierarchy of inclusive blocks. In this case, the 4-component (block 4) is fully embedded in the 3-component (block 3) and the 3-component in the 2-component (block 1). Such components are maximal and unique, and they always stack in this way, but any given network may have many different *k*-components for a given value of *k*, each of which will be part of a distinct inclusive hierarchy.

Cohesive groups need not be partitions of the nodes in the network. Not only can they be subgroups that are embedded in one another, but also two or more *k*-components may overlap without one group being a subset of the other.[1] Examples are found in White and Harary (2001) and Moody and White (2003) but not in fig. 2. Because the *k*-components of a network do not always form a partition when $k > 2$, none of the partitioning methods in network analysis (such as blockmodeling of structural positions in a network) is capable of identifying structurally cohesive sets that overlap or that embed one such set in another. The methods of analysis that are required for analysis of structural cohesion require distinct analytic techniques, those of cohesive blocking (Moody and White 2003).

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[1] Disconnected networks, of course, will have more than one component, and connected networks may have more than one 2- or higher-level connectivity component. Unlike simple (1-) components, however, two *k*-components with $k > 1$ within the same network need not be mutually exclusive, and can have as many as $k - 1$ nodes in common.

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