Discovering City-Curve Oscillations: Historical Dynamics as a Reactive System for China, 900 CE to the Present

Author #1* Author #2†
Author #3‡

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Abstract

For the largest world cities, in 24 historical periods over the last millennium, in each major urban region of the world—China, Mid-Eurasia, Europe, Japan, and North America—we fit city size distributions to the q-entropy model (Tsallis q) that asserts constrained variation in a continuous and closely bounded form between two poles that depart in different ways from entropy. Departures from randomness ($q=1$) are modeled in one of two directions, one for asymptotic tendencies toward power-law tails in which the excess of people in cities of size x or more is ($q>1$)-scale-proportional to their size (and the larger q, the thicker the power-law tail) if they are above a certain crossover size, while below the crossover frequencies asymptote toward entropy or random variation. The opposite tendency modeled ($q<1$) is asymptotic decay toward a linear decline in number of people in larger cities. The Tsallis entropy model provides a baseline for studying complex interactive systems and outperforms Zipf's and power-law models, with considerably better R2 fits. It allows us to fit key parameters with which to theorize population history and urban demography based on much more useful, interpretable, and informative measures that encapsulate differences and changes over the entire range of city sizes rather than just the largest cities. Here, in the first of our family of regional and interregional analyses, we study the population history and urban demography of China. Use of Spss and Excel Solver for nonlinear regression packages, excluding two periods with insufficient data, shows convergent fits of all the city size curves to the q-model with very high resolution (R2$\sim$.98).

We use a Galilean method of small-differences to show a way to understand and verify the discovery of oscillations in city systems during the period of globalization from 900 CE to the present era. Fitted parameters yield accurate historical descriptors of major dimensions of incremental and episodic structural changes in city systems. Among these dimensions are changes in the shapes of the size curves (q and κ) and estimates of the total urban population size (Y0). There is reason to believe that the Y0 percent urban estimates for China over the last millennium are accurate, and, even in recent decades, better than the Chinese census estimates. Matching the model estimates against total population, which is accurately known for China, the model parameters provide accurate quantitative data for modeling historical dynamics that indicates consistency with secular cycles of interactive population growth and conflict-related decline as studied by Turchin (2005, 2006).

Acknowledgements: We thank Ernesto Pinheiro Borges for setting us right on many occasions regarding Tsallis entropy and the mathematics of the fitting methods employed, and for explanations of elements of his thesis. We thank Constantino Tsallis for introducing us to the theory and methods of q-entropy and Peter Turchin and Donald Saari for providing guidelines on what is needed for analysis of historical dynamics. We are solely responsible, however, for our errors in any of these areas. We are grateful to Céline Rozenblat for providing us with the first dataset on historical urban populations, that of Chandler and Fox (1974), to Chris Chase-Dunn for providing the Chandler (1987) data, and to George Modelski for very helpful commentary.

Keywords: Zipf's law, city sizes, q-entropic model, longitudinal, historical data, China, Spss, Excel, nonlinear regression, complex social systems, multiplier effects, secular population cycles, sociopolitical instability, dynamics, near-equilibrium, edge of chaos

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INTRODUCTION

The project from which this paper issues is one designed to give quantitative historians and social scientists a new set of variables for research on historical dynamics. The initial goal is show how the construction of these variables is solidly grounded on measurement that uses scaling techniques based on well-tested concepts in the science of complexity. In this first paper we focus on *discovery*, that is, new, robust, and reliable measures that can be shown to be relevant to understanding historical processes. In this case the processes are those of the demographics of a city system in which market systems evolved initially in East Asia and impacted subsequent evolution of the Eurasian world system of interlinked urban economies.

Discoveries are rarely accepted until the evidence is clearly seen and verified. Explanations follow after. Some never follow at all. Typically, since Zipf's (1949) "law" as a popularization of Auerbach's (1913) discovery of urban power laws for city sizes, urban concentration research has simply disregarded all but the tails of city size distributions. There is no agreed upon explanation for "Zipf's law" (Krugman 1996) but only empty or elusive claims of invariance.

Discoveries are easily received by some audiences, and with difficulty by others, so we keep our presentation simple. Our discovery is that of an historical *q*-entropy metric for city size curves that has reliable and interpretable properties. The metric model (1) measures distributional changes in city sizes of successive historical periods along three comparable dimensions, (2) helps identify historical periods in terms of how their city size distributions differ in these dimensions, (3) takes into account the full range of city sizes beyond the power law tails of the distributions, (4) estimates where a crossover takes place to a power law in the distribution, if at all.[1] We also explore (5) estimates of total urban population even in the absence of censuses for the smaller cities. Our curves as new historical "objects" of study have parameters that are interpretable in terms of (a) the mathematical model of *q*-entropy, (b) a standard variety of complexity theory, and (c) linkage to urban processes relevant to the study of historical dynamics.

Tufte (1990:18-22) describes how in 1610-1612 Galileo's method of recording small differences resolved disputation about the existence and behavior of sunspots following 200 years of naked-eye viewing in Athens, China, Japan and Russia. "It was difficult for Europeans to see sunspots at all because Aristotle had said that celestial bodies were perfect and without blemish, a fancy that became official Church doctrine in the middle ages." His method was to focus the telescope on the sun, let the image shine on a piece of paper, draw the outline of the sun, note the day and time and the orientation of the paper, and then trace in minute detail the locations of the sunspots. The fact that regular configurations were seen to rotate as days went by was the convincing and verifiable evidence. It took another three and ¾ centuries to understand the thermonuclear explosions that generated them and their regularities. It is now understood, for example, that there are there are regular 11 year periodicities in solar energy bursts, and why.

Using the data on city sizes carefully and exhaustively collected by Chandler (1987) for all the larger world cities down to certain rank-size, over successive fifty-year intervals, enables us to use the method of small differences in subcontinental regions such as that for China to

[1] The term crossover is used for the point at which the continuous *q*-entropy function is in the middle of a gradual asymptotic shift from the curve for the body that is consistent with an asymptotic approach to a power law in the tail.

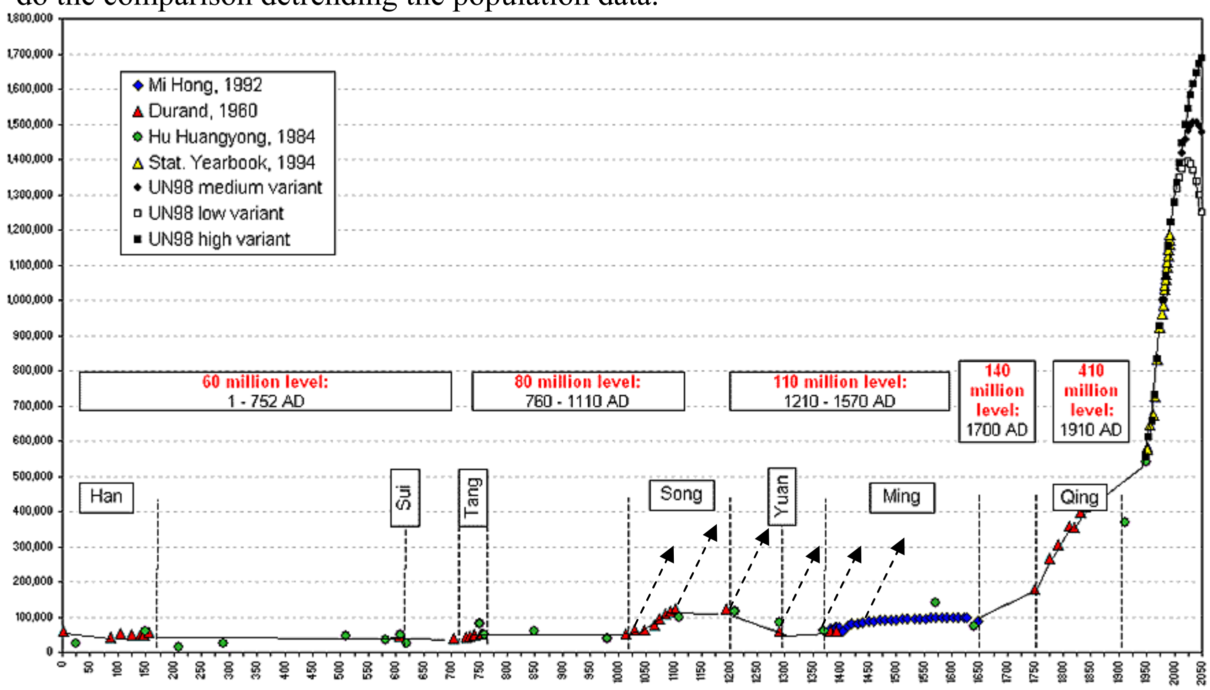
plot patterns of change in the curves of urban sizes. Here, with the method of small differences, we instantiate our discoveries using the Chinese city data of Chandler.[2]

The patterns in our city size metric data and Galileo's sunspot behaviors have, at first sight, the same kind of irregularities, recurrences, and apparent complexity. Apparent complexity in a new discovery may well take time to understand. This is the second of a series of articles on the subject of cities.[3]

MAPPING THE DISCOVERY IN SMALL-DIFFERENCES

An Empirical Example of the Discovery

We exemplify our discoveries here for Chinese cities, 900 CE – 1970. As background, growth in China's population in that period is shown in Figure 1 (adapted from Heilig 1999). The dotted arrows are indicators that changes in total Chinese population (rural and urban) precede changes in our city metric by 50-100 years. The same is true for later periods, but the total population rises vertically while the city metric is confined between 0 and 3, so to do the comparison detrending the population data.



Historical Time, 0 CE - forward

Figure 1: China's Population (from Heilig 1999: pop_21_m.htm, with symbols for data sources)

The historical structure of q in China's cities, 900-1970 CE: Figure 2(a)

The history of cities is not a smooth progression. To make visually evident major kinds of changes, the curves that are graphed in Figure 2(a) are not the raw data *per se* but those curves that precisely fit the raw data to the three parameters that govern the observed *mix* of the exponentially random with power-law or linear tendencies. These are: q for extent and direction of departure from a baseline (q=1, an *exponential* curve) of randomness, Y0 an

[2] While we have modeled all the major world regions with reliable results, the results are more useful and more easily evaluated both region by region and in studying time-lag interactions between regions.

[3] The first is a tutorial that contains a section on our measurement methods (White, Tambayong, and Keyžar 2006a). The third (2006b) will place the present application for historical China in the larger context of the Eurasian world system (Modelski and Thompson 1996) and its interactions in this millennial time frame of globalization.

intercept term representing the total *urban population*, to which each curve asymptotes horizontally, and κ as a scaling unit affecting the crossover either to the nonexponential asymptote of a *power-law* where $q>1$ or to *linear* tendencies in the case where $q<1$. Y0 in our study is an estimate of the total urban population in a given historical period. The mathematical model and statistical theory of these mixtures is that of q-entropy (Tsallis 2004).

Figure 2(a), at roughly 50 year intervals over a millennium, gives a visual summary of the stabilities and changes in the new "object" of historical study: fitted distribution curves for cumulative city sizes over these years. The x axis shows logged bin sizes for thousands of city residents. The bin sizes for x are successive multiples of $\sqrt[3]{2}$, starting with 31.6K. The multiple $\sqrt[3]{2}$ is optimized for the sake of precision,[4] but the binning is robust in that changes in the multiplier will not affect the results so long as there are sufficient bins with cities in each bin over a sufficiently long periods to do the scaling. The y axis is the population in cities of size x or greater (in thousands or millions) at a given time t for each dataset. Each dataset is represented by a curve. Extension of the fitting lines to the y axis is incomplete because we lack data on city sizes under 40,000, a limit imposed by Chandler for data on comparable numbers of largest cities in each period. The height of different lines represents historical shifts in population numbers. The figure is a log-log graph in which a Zipfian power law for any of these data would show as a straight line (q=1.5). Out of 24 periods there are three straight lines, for the years 1450, 1600, and 1970 (upper left), and five that are nearly straight: 1000 (lower right), 1150, 1500, 1575, and 1925. We hold judgment as to whether the Zipfian for these data is in any sense "optimal."

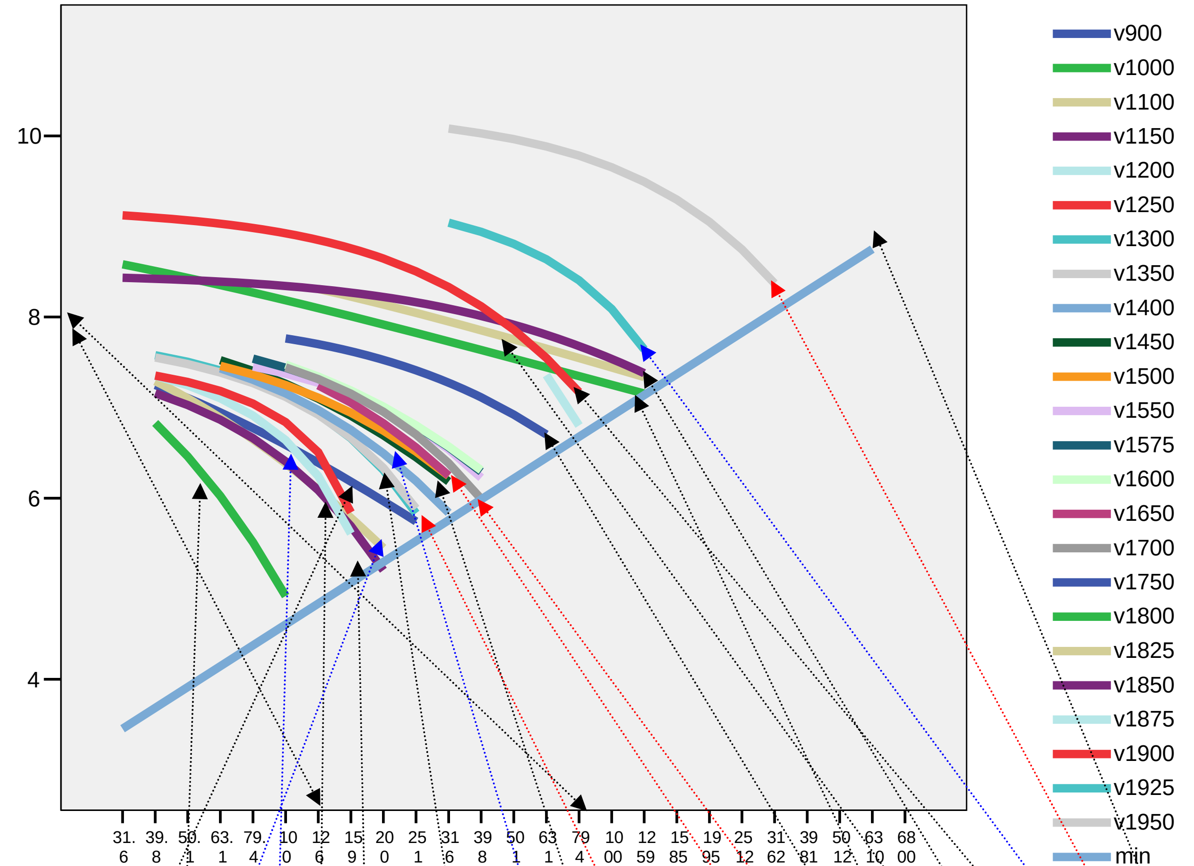
Figure 2(b) shows the q metric of extent and direction of departure from the null hypothesis of random variation in city sizes, where q=1 indicates no departure. For $q>1$ these curves asymptote for the larger cities to a power-law with slope 1/(1-q). Arrows upward from the dates at the top of 2b show links to the city curves in Figure 2(a). The curves change order irregularly but are bunched rather than random. We see rises and falls in q that are affected by events such as invasions, war, or periods of innovation affecting Chinese cities and trade networks. Such events impacted not only the economy of East Asia but, given the primacy of China, other parts of Eurasia, including the Middle East and Europe. Many of the slope changes take place about 100 years after rises or falls not just of city populations but of the total population numbers shown in the Figure 1.

History or randomness in q: The runs test for China's cities*, *900-1970 CE (p=.06)

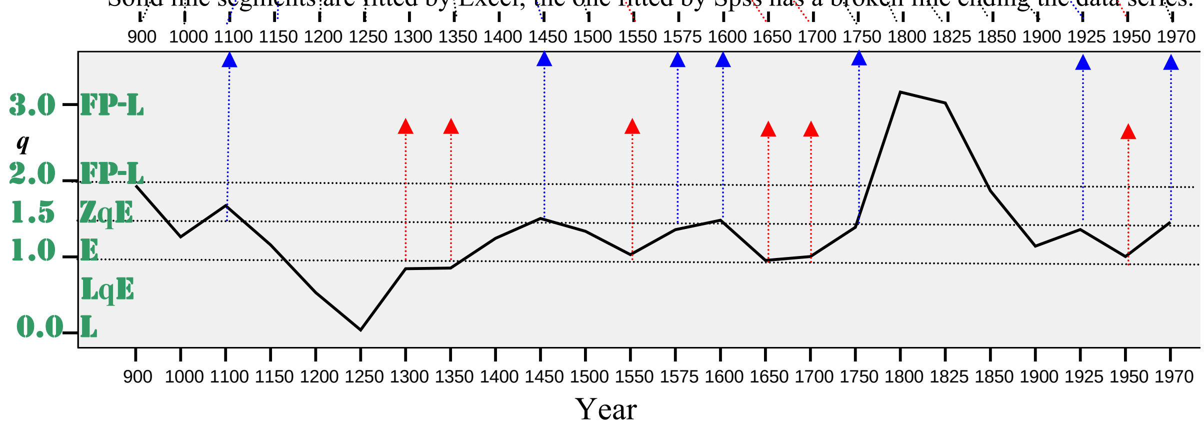
China is a good case in which to test the appropriateness of the q-entropy model for city sizes. The city size data over the last millennium are well documented and coded by Chandler (1987), who is considered reliable. The values of q for the 24 periods in Figure 2b distribute with a mean of 1.32 but with both skewness and kurtosis significant ($p<.05$) and outliers of $q>3$. Are the variations in Figure 2 real and meaningful, or random? A preliminary test runs test (Bradley 1968) is made by dividing the 24 observations into those above and below the mean. This division produces historical runs (Q-periods) with sequentially all higher or lower values than the mean of q. Is their average length or number random? Eight runs occur, fewer than expected at random, with probability p=.06. A runs test at the median gives the same result. A "best" cut at 1.03 gives seven runs and p=.05.

[4] If the primate city that occupies the largest bin is twice the size of the next city, as expected with a Zipfian distribution, bin placements then differ by three, as in the raw data graphed in Figure 5. This bin discontinuity led us to omit the primate city from the fitting in each period for both the q-entropy model and power-law fits.

Even over this small N of 24 periods, *q* thus varies systematically. As we shall see further on, *q* is highly informative in measuring structural aspects of historical processes.



(a) City sizes in 1000s, each bin a number in 1000s of people in cities of at least this size
Solid line segments are fitted by Excel; the one fitted by Spss has a broken line ending the data series.



(b) Parameter *q* in temporal succession, measuring extent and direction of departure from the null hypothesis of random city-size variation. ***Key***: L=Linear in raw data; L*q*E *q*-exponential; E=exponential; Z*q*E=*q*-exponential with Zipfian tail; FP-L= flat power-law. Blue dotted arrows help identify curves with *q*~1, red arrows identify Zipfian *q*~1.5 curves. The translations of *q* values in these terms into homogenous or heterogeneous portions of the city distribution is given in the text.

Figure 2: Variations in city curves for China, 900-1970 (best viewed in color)

THE SCALING MODEL

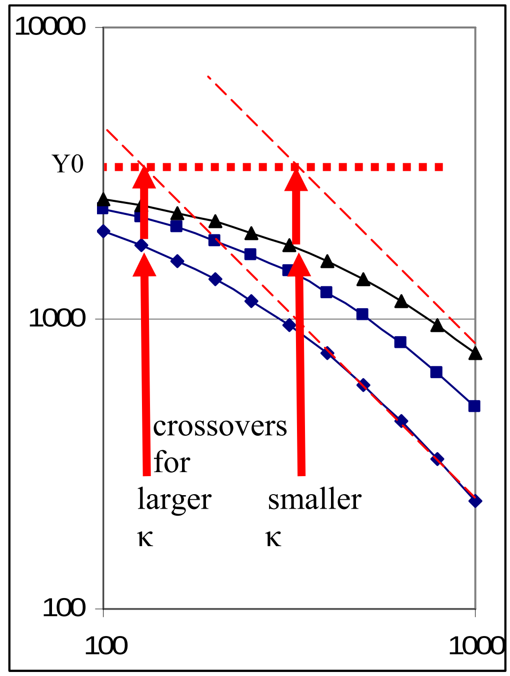
The q-entropy model: variations in q

The generative mathematical model of Tsallis q-entropy (Tsallis 1988), in which q (≥ 0) is a positive real parameter, asserts that compared to the null hypothesis, for which q=1, departures in the direction where q>1 take the form of proportionality effects such as simple attraction mechanisms that occur above a crossover in the distribution, as reflected in shift to a power-law tail. Deviations from the null case in the other direction, where q<1, shift from exponential distributions to linear (q=0). The scaling equation for Tsallis entropy is

$$Y_q \equiv Y0\,[1-(1-q)\,x/\kappa]^{1/(1-q)} \qquad (1), \text{ where}$$
$$e_q^{x'} \equiv \quad [1-(1-q)\quad x'\,]^{1/(1-q)} \qquad \text{and } e_{q=1}^{x'} = e^{x'} \qquad (2)$$

is the *q-exponential.* We will take up each case in the q-entropy model in turn, applied here to a complementary cumulative frequency distribution (CCFD): q=0, 1, 1.5, and larger, with examples drawn from those fitted in Figure 2(a). In our study the variable x is city size but Y, fitted by $Y_q(x)$, is the number of people in cities of size x or greater. Y_q, that is, is a complementary cumulative frequency distribution (CCFD) or Pareto (1896) distribution, in which case a power law has a slope one-greater-than that of the noncumulative distribution for the same data. Y0 is a parameter that represents *the total urban population* and *not the total population P* of the region from which the cities are sampled. If we know the minimal city size for an appropriate definition of city then Y0 is defined independently of q, so it does not need to be estimated. But if Y0 is unknown then part of the fitting of equation (1) will solve for Y0 along with q and κ. A check on a correct fit in this case is that (i) Y0 *must be smaller* than P for the region and in fact represent the urban portion of P. Further, (ii) Y0 *must be bigger* than any of the city size bins that are estimated. This gives a sense in which the fitting of q-entropy represents real physical phenomena with physical constraints. In this context, Y0 represents the population in those human settlements that interact in such a way that, if they are found to have nonlinear or proportionality effects, they are part of the "system" in which such effects occur. That is, we would not expect settlements of size 1 (hermits) or even size 1,000 (rural villages) to be part of the "urban system" in which there are proportionality effects or power law tendencies for size distribution.

The parameter q reflects a real numeric scale of the extent to which there are nonrandom interactions (if q departs from 1) in the system of interaction, either linear or nonlinear. And why should we consider cities to constitute systems of interaction? Here again, the idea is one of a physical system: cities are settlement units that have a division of labor with respect to other such units (internally and externally) such that they are dependent on exchange interactions with other cities. And finally, κ is a real-valued parameter that in combination with Y0 and q reflects the crossover for population sizes at which interactions emerge as departures from randomness (with linear or nonlinear power-law tendencies). For power-law tails, the crossover occurs earlier the smaller κ and (q-1) and is determined by the inverse of their product. Thus for example, while κ=1, Y0 very large, and q=1.5 would define a very large Y0 population distributed in cities so as to approximate a Zipfian power law, this is unlikely to occur empirically because it implies that even the smallest cities behave as if they growing at a rate proportional to their size, and the Zipfian distribution covers the whole range of cities. Empirically, the crossover coefficient κ for city size is much larger than 1. But it is unlikely as well that κ would be anywhere near the magnitude of Y0 because that would imply that the city sizes behave as if they were perfectly random, and lacking a crossover to a power law.



The graphical inset shows the effect of taking the fitted size distribution for 1750 (the middle curve), which ran from bin size 100K to size 500.1K, extending the curve to run to a size of 100K for purely aesthetic reasons, and then creating curves above and below that vary only by changing kappa (κ) from 343.4 in the middle curve to 200 in the upper curve and to 500 in the lower curve. What is unchanged is the asymptote (the dotted horizontal) to the same Y0, that of 3,098K. Increasing κ creates the lower curve, which is more bent, and moves the crossover (upward arrow) for the asymptote to power law in the tail toward the Y axis. Because q is constant in this example each curve has exactly the same asymptote to power law slope. The larger κ reduces the cumulative number of people living in the larger cities. The crossovers in each case are located by the intercept of the asymptotic power-law slope (the dashed lines) and the Y0 asymptote. Assuming that Y0 asymptotes to a constant value at 10K, in this example multiplying κ by $^3/_2$ has the effect of reducing the log-location of the crossover by $^3/_4$ths because the limit on the largest city size is also reduced to, say, $^8/_9$th, enlarging κ relative to that line by $^9/_8$ (thus $^2/_3 \cdot ^9/_8 = ^3/_4$). The mathematics of q-entropy used here is multiplicative.

The q-entropy model explicated for parameters q, κ, and Y0

At q=0 (as shown for **L** in Figure 2b), Y_q = Y0 (1-x/κ) is a simple straight line where Y0 is the intercept and -Y0/κ the slope. In the direction of deviations from the null hypothesis where q<1, the Y_q curves change toward this linear model, $Y_{q=0} \equiv Y0\text{-}xY0/\kappa$. The year 1250 is an example from Figure 2(a) with q≈0 and -Y0/κ≈-11, for a log slope -2.35. The curve has a long straight tail that asymptotes to slope -2.35/1.[5] It bends toward an exponential for smaller cities because q is slightly greater than 0. Estimates are .03 < q < .05 and 185 < κ < 190. In a growth model this would correspond to a constant absolute growth increment regardless of city size.

At q=1, Y_q is defined by the fact that $[1\text{-}(1\text{-}q)\, x/\kappa]^{1/(1-q)}$ converges to the exponential exp(x) as q→1, so that $Y_{q=1} \equiv e^x$. For 0≤q<1 there is no crossover to a power law because distributions that vary from straight lines at q=0 to exponential curves of complete independence at q=1 are not skewed by proportionality effects. The parameter value of q=1.0 that defines the exponential curve for the null hypothesis is consistent with the convergence of Y_q as q→1 to the curve $Y_{q=1}=Y0e^{x/\kappa}$, with its exponential parameters given by Y0 and κ. In a growth model this would correspond to a distribution of growth rates independent of city size. The curves with q≈1 in Figure 2(a), for years 1150, 1300, 1350, 1550, 1650, 1700, and 1950, all have the typical curvature of the exponential as drawn in log-log. The curve for year 1150 is easily visible in dark purple in Figure 2(a). The parameter values of κ and Y0 for these years are in different ratios, so they have different

[5] For q>1, the tail asymptotes to a log-log slope -1/(q-1) but as q goes from 1 to 0, the tail asymptotes to a linear slope -Y0/κ.

curvature, just as an exponential growth curve x_0e^{kt} will have a different growth rate k. Larger κ flatten the $e^{x/\kappa}$ distribution, for example, as seen in comparing 1700 to 1150.

For $q>1$, the asymptote toward a power-law tail with some scaling exponent occurs after a crossover involving other parameters. Here the model applies to the assertion that large cities above the crossover tend asymptotically to grow q-scale-proportionally to their size. At $q=1.5$ the asymptotic slope of Y_q becomes Zipfian, following the rank-size rule (Zipf 1949) of frequency proportional to rank of $^1/_1$, $^1/_2$, $^1/_3$, …, $^1/_r$. The equivalent cumulative (CCFD) slope of -2=1/(1-q) is shown by the bidirected arrow in Figure 2(a).[6] The Zipfian has linear-proportionality effects. For q slightly below 1.5 the proportionality effects are sublinear. The years 1000, 1450, 1575, 1600, 1750 (possibly 1875), 1925, and 1970 are examples of $q \approx 1.5$. In a growth model this would reflect growth linearly proportional to city size. For q slightly above 1.5 there are superlinear proportionality effects. As $q \rightarrow 3$ the asymptotic slope becomes much flatter than Zipfian, as for example in 1800 and 1825. In a growth model this would reflect growth—including migration or mortality effects of conflict—superlinearly proportional to size. Close examination of Figure 2(a) will reveal differences in curvature of city size distribution if one keeps in mind that differences in Y0 reflect where the distribution asymptotes to a horizontal line that intercepts with the Y axis, and that greater or lesser curvature for two curves that have the same X and Y coordinate intercepts will be a function of κ. Lower κ makes for straighter lines, becoming straighter in the log-log graph as $\kappa \rightarrow 1$. Higher κ makes the curves bow upward.

ESTIMATION METHODS

The q-exponential curve models nonlinear dimensionality typical of complexity. Fitting equation (1) for the parameters of q-exponentials, however, is simple. Getting estimates in Spss with the Chandler data is a matter of using Analyze/Regression/Nonlinear to fit the formula Y0*(1-(1-q)*x/k)**(1/(1-q)). We entered Y0, q and k into the parameters window with initial values q=1.5, k=72 and Y0=2635. The only limits set initially were q > .01, q < 4, k > 5, and Y0 > 0. The values of x were the population sizes, an actual variable in the Spss spreadsheet that indexes the population size bins as shown on the x of Figure 2(a). Since we binned our data in increments of $\sqrt[3]{2}$ we named our x variable "binlogged." They take values 31.6, 39.8, 50.1, 63.1, 79.4, 100, 126, 159, 200, … up to 6,800 thousand (6.8 million) people. The dependent variable is specified from among the spreadsheet variable names: c900, for example, was the variable for the cumulative numbers of people living in cities of at least the sizes 31.6, 39.8, 50.1 thousand, and so forth. The loss function was set to the sum of squared residuals.[7] Choice of initial values made a difference to the outcome only when we reached large population sizes in the later years, where we reset the Y0 constraint to Y0 > 2000 in order to get convergence.

To crosscheck the Spss estimates we did nonlinear regression in Excel, using Tools/Add-ins/Solver Add-in ("Ok") and then Tools/Solver (see White, Tambayong and Kejžar 2006a). We specified the same equations, constraints, initial values, and loss function. The results of independent Spss and two Excel runs for China are shown in Figure 3. They give nearly the same results for q (R=.966, for years 900-1970), with means ca. 1.33.[8] Correlations among κ

[6] In a cumulative decreasing distribution, a Zipfian slope corresponds to -2; if increasing, then +2.

[7] Converting our datasets to CDF probability distributions normalized on Y0, it should be possible to do maximal likelihood estimations. This will be possible if there is evidence that the Y0 values are reliable.

[8] In our initial computation there was a discrepancy for the year 1850 in how the data were right edited, giving R^2 =.92 for the period 900-1825 but R^2 =.81 if the series is extended to 1950. When corrected the two 1850 q-

(kappas) and between the Y0 values from the Spss and Excel results are even higher (R=.990 and R=.995). The nonlinear regression results are convergent at a level of precision where we can be sure that the variations we see in Figure 3 come from best-fits for well defined equations and tested algorithms.[9]

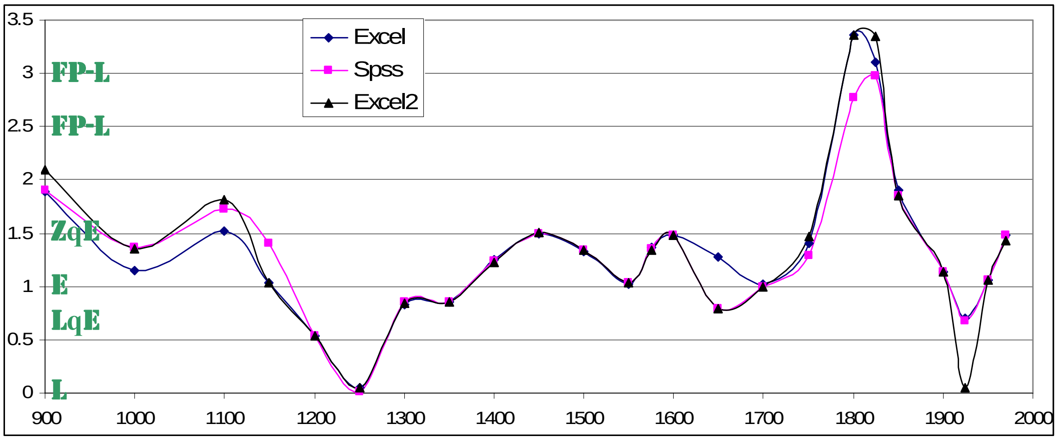


Figure 3: Estimating *q* from Spss and from Excel– China 900-1970

Bootstrapping confidence intervals

Spss allows us to bootstrap estimates of the standard error for our *q*-entropy parameters, and as can be seen from the example below: for c1650, the standard error for *q* is quite low (.094 on an estimate of .795), as it is for most of our estimates.

Table 1: Example of bootstrapped parameter estimates for 1650

Parameter Estimates

			95% Confidence Interval		95% Trimmed Range	
Parameter	Estimate	Std. Error	Lower Bound	Upper Bound	Lower Bound	Upper Bound
q	.795	.094	.608	.983	.795	.795
k	229.307	6.854	215.592	243.022	229.307	229.307
Y	2471.785	3.307	2465.167	2478.403	2471.785	2471.785

a. Based on 60 samples.

b. Loss function value equals 4161.644.

HYPOTHESES AND TESTING

Five hypotheses are tested. The first two deal with the appropriateness of different models. The third deals with the parameters of the model that we find to be appropriate, and maps the discovery of the model fit to the historical phenomena. The last two hypotheses are derived from the mathematics of the model and tested against the historical data.

Hypothesis 1: The *q*-entropy model is appropriate for city-size distributions, esp. the body.
Hypothesis 2: Power laws are not appropriate for city-size distributions, beyond the tails, and while the log-normal distribution is appropriate if growth rates are uniform for cities of

values converged and the correlation changed to R^2 =.92 for the period 900-1850 and R^2 =.86 to 1950. Examining 1950 we found that our run had not converged and that our initializations of Y and *k* were too low, so we multiplied *k* by 100 and Y by 10. Those *q*-values for 1950 then converged and the *q*-series converged to R^2 =.92 for the period 900-1950. Continuing in this manner we would have 1650 and 1800 converge perfectly, and similarly for 1000, 1100 and 1150. We also had to adjust the constraints for excel on 1970 so that k>665, Y>200000 and initialize q=1.5.

[9] We note that Spss was less divergent subject to variation in initialization values.

different sizes, its cumulative distribution does not fit historical cities.
Hypothesis 3: *q*, κ, and Y0 are historically interpretable.
Hypothesis 4: There is an interactive dynamic between population cycles and *q*-oscillations in which *low-q*(<1) is in the down cycle, caused by a peak population (thus scarcity) and subsequent decline (presumably by depopulation in leading cities and collapse of the power-law asymptote). In terms of time lag: population peak → conflict and *q* down(<1) (pop. down) → lower conflict → population growth. The mathematical derivation will require an understanding of the specific parameters.
Hypothesis 5: In the interactive dynamic between population cycles and *q*-oscillations, κ should play a role along with *q* in the population recovery phase because it is involved in the *extent* of the reemergent power law. Again, the mathematical derivation will require an understanding of the specific parameters.

Evaluation of Hypothesis 1: Goodness-of-Fit, Runs Test, Commensurability Tests

(a) Goodness-of-fit for q-curve and China cities data, 900-1970 CE (mean R^2≈0.97)

Hypothesis 1, that the *q*-entropy model is appropriate for city-size distributions, is most directly evaluated by goodness-of-fit tests. Plotted in Figure 4 are the goodness-of-fit correlations between actual and fitted data for the 24 fitted time periods. Statistical analysis of the Chinese cities data series shows convergent fit to the *q*-entropy model. (China in 1914 lacked top-75 world cities outside a single bin and 1875 beyond two bins and both were excluded from this test.) The average fit is R=.987 (R^2=.984), with a std.dev. of .018 and no kurtosis. Skewness is significant because of proximity to the R^2 limit of 1.0, i.e., clustering toward 1.0, and with the null probability p=.01. There is a slight trend toward lower goodness-of-fit in some of the later years. Higher fit is partly a reflection of Chandler's collection of more data for 1800 and 1900 than for earlier periods. Disruptions may also lower the fit to the *q*-exponential model. Post-WW I China (1850) following the Opium Wars (1830-) does not fit well.

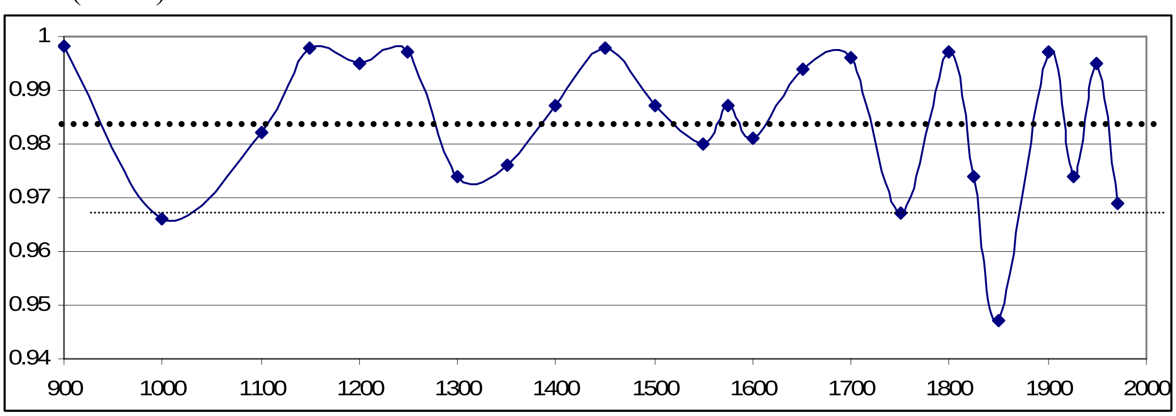


Figure 4: Variation in R^2 fit for *q* to the *q*-entropy model – China 900-1970
Key: Mean value for runs test shown by dotted line.

(b) Runs test for data bias

Eleven values in Figure 4 are below the mean and median and 13 at or above; there are 16 runs above or below the mean. For these variations we cannot reject the null hypothesis (p=.28). No historical runs can be said to be particularly worse or better in goodness-of-fit. This speaks well for uniform quality of the Chandler (1987) data.

(c) Evidence of Commensurability

Hypothesis 1 is strongly supported. A further test is that of commensurate-ordering of (A) actual total population, (B) Y0-total urban population estimate, (C) lowest bin estimates, and

(D) κ. As shown in Table 2, all these are positively correlated and form a single factor with 87% common variance (with the communality loadings shown in the table). All these variables are weakly correlated to the slope 1/(1-q) of the power-law tendencies (.35) but not with 1/(1-q) for q<1. The ratios between the pairs of theoretically ordered variables—A, B, C, D—are shown in columns 2, 4, and 6 of Table 3 as ratios (1) A/B, (2) B/C, and (3) C/D.

Table 2: Correlations among the commensurate-ordering variables in Table 3

	Pop	Y0	31.6K	Communalities
Total Chinese Population				.88
Y0 Estimate	.75**			.95
Bin Estimate at 31.6K	.81**	.96**		.97
K	.70**	.81**	.90**	.91

* p <.05 ** p < .01

Table 3: Commensurate-ordering test of Population, Y0, Bin estimates, and Kappa

Year	A China Pop.	**AB Ratio Y0/Pop. %Urban**	B Y0	**BC Ratio Asymptotic Y0 Slope**	**C** Bin Estimate at 31.6	**CD Ratio Power Law**	D Kappa κ	*q*	(-)Slope $\frac{1}{q-1}$
900	13,000	.295	3829	.422	1615	.014	29.1	1.90	1.11
1000	34,500	.268	9236	.143	1322	.009	15.6	1.37	2.69
900	70,000	**.042**	3829	**.422**	1615	**.014**	29.1	1.90	1.11
1000	80,000	**.115**	9236	**.143**	1322	**.009**	15.6	1.37	**2.69**
1100	98,500	**.070**	6883	**.264**	1819	**.007**	13.2	1.72	1.38
1150	85,000	**.025**	2112	**.673**	1422	**.056**	100.6	1.40	2.50
1200	110,000	**.020**	2170	**.771**	1673	**.079**	132.5	.54	
1250	55,500	**.036**	1986	**.831**	1650	**.113**	185.7	.02	< conflict
1300	79,800	**.032**	2555	**.809**	2067	**.074**	194.0	.85	
1350	87,000	**.029**	2497	**.807**	2016	**.074**	151.2	.85	< conflict
1400	88,000	**.032**	2836	**.765**	2171	**.053**	114.4	1.24	4.18
1450	100,000	**.031**	3086	**.762**	2351	**.047**	109.6	1.50	2.01
1500	115,000	**.022**	2541	**.822**	2089	**.075**	178.4	1.34	**2.96**
1550	145,000	**.016**	2366	**.883**	2090	**.122**	253.7	1.04	28.57
1575	164,000	**.017**	2861	**.841**	2405	**.073**	177.1	1.35	2.87
1600	170,000	**.019**	3228	**.809**	2612	**.054**	140.7	1.48	2.07
1650	88,500	**.028**	2471	**.870**	2149	**.107**	229.4	.80	conflict
1700	100,600	**.028**	2782	**.857**	2385	**.086**	205.2	1.00	
1750	183,500	**.017**	3097	**.914**	2830	**.121**	343.4	1.29	**3.51**
1800	300,000	**.030**	9085	**.587**	5330	**.006**	29.6	2.77	.56
1825	370,000	**.019**	7014	**.814**	5710	**.022**	125.8	2.97	.51
1850	436,300	**.011**	4804	**.957**	4595	**.152**	696.5	1.85	1.18
1875	362,000	*	.	**.**	.	**.**	.	.	< conflict
1900	396,430	**.025**	10048	**.911**	9150	**.037**	339.7	1.14	7.14
1925	450,000	**.034**	15269	**.940**	14359	**.035**	508.2	1.39	2.56
1950	550,000	**.055**+	30394	**_.978**	10447	**.076**	2253.6	1.06	16.67
1970	830,000	**.024**	20000	**.929**	29727	**.036**	666.0	1.49	2.02

* 1875 is a population low after the Opium wars and Taipei Rebellion, but had too few world cities relative to other parts of the world for reliability estimates.

+ (from Table 3) The Y0 of 30.394 Million urban residents in 1950 is less than half the 72 million urban population of official statistics, but this may be due to different cutoffs for what size is considered urban. Heilig (1999: trend_30.htm) reports: "According to official statistics, China's urban population was about 72 million in 1952; in 1997 it was estimated at 370 million (see Table 3). China's urban population has more than quadrupled since the early 1950s, whereas its rural population has only increased from 503 to 866 million. Despite this enormous increase in urban population, China has essentially remained a rural society. More than two-thirds (70%) of the population is still classified as rural."

In theory the total population (P) of China should be greater than the *q*-entropy estimates of Y0 since Y0 is the total urban population, Y0 should be greater than the cumulative population in the lowest bin sizes estimate in our Figure 2(a), those of 316K persons or more, and the lowest bin size estimate should be larger than κ. Columns 1, 3, 5, and 7 of Table 3 contain these variables and provide for comparisons among them by taking their ratios.

(d) ***Commensurability of Chinese Population, Y0, Bin Estimate, and*** **κ**

The AB ratio (ratio 1: Y0/Total China population) should estimate the **urban proportion** of the population. Multiplying by 100, it is an estimate of % Urban. It is correlated with *q*, as shown in Table 4, and negatively with the other ratios. The **% Urban** trend for ratio 1 shows very slight increase over time, consistent with China's tendency to remain a rural society until recently. We have two sets of estimates for 900 and 1000, the second of which (see Figure 1) have urban proportions of 4.7% and 11.5%, which are probably more accurate than the first. Using the higher total population estimates, the urbanization of 1000 is estimated here at 11.5%, the time of emergence of national markets based on paper money and printing, vast production of coal and iron, and huge armies (Modelski and Thompson 1996:164). This figure is not reached again until after 1970 (the last year of the Chandler data series).

Table 4: Correlations among the commensurate-ordering ratios in Table 3

	AB ratio 1 % Urban	BC ratio 2 Y0 Slope	CD ratio 3 PowerLaw
AB Ratio 1-Urban proportion: Pop/Y0			
BC Ratio 2-Y0 approach slope: Y0/Pop estimate bin31.6	-.75**		
CD Ratio 3-Power law: Pop estimate bin 31.6K/κ	-.49*	.69**	
κ		.51*	
Total Chinese Population		.40*	
QPopulationTrend		.47*	
PopQDetrended		-.39*	
LCPDetrend 50years before		-.47*	
LCPDetrend 25years before		-.50*	
LCPDetrend		-.32(ns)	
LCPDetrend 25years after		(n.s.)	
LCPDetrend 50years after		(n.s.)	
q			-.45*

* p <.05 ** p < .01

(e) ***Commensurability Ratios***

The BC ratio 2 (Y0 estimated urban population/population estimate in lower bin 31.6K) and the CD ratio 3 (pop. estimate bin 31.6/parameter κ) are also consistent with theoretical expectations in that they are between 0 and 1. As shown in Table 4, they correlate with one another, and both correlate negatively with ratio 1. The **Y0 approach slope** (ratio-2) is so called because of the logic of the *q*-entropy curves and the parameter κ that governs not only the crossover to a power-law tail, but the asymptotic approach to Y0. It is correlated with κ, so that larger κ decreases the slope from the asymptote Y0 to the crossover to a power-law tail (i.e., the approach gradient to the approximate total urban population Y0). Thus, a higher BC ratio is a flatter slope with fewer cities in the lower bins and concomitantly more in the upper bins. Not surprisingly, then, it correlates with total population (R=.40), ratio 3 (R=.69), a steeper population trend as fitted to *q*-exponential growth (QPopulationTrend, R=+.47). Surprisingly, it is *inversely* correlated to *q*-detrended total population (PopQDetrended, R=-.32), so that detrended overpopulation entails high population concentrations in large cities. Because of this correlation, we thought that the **Y0 approach slope** could be important in temporal relationships to our variables for logged Chinese detrended population oscillations (LCPDtrend) which we computed in lagged time intervals for use with dynamical modeling (i.e., antecedents or consequences of total population changes). Table 4 shows that *detrended population rise 25 (or 50) years prior correlates with* ***decline in the Y0 approach slope*** and presumably greater numbers in the smaller city size bins.

Ratio 3, with κ in its denominator, is inversely correlated with *q*, which in turn is inversely correlated to κ. This is consistent with less variation in the lower size 31.6K bin estimates than in *q* or κ. This ratio provides a separate measure for the asymptotic power-law slope that needs further study. We assigned it a tentative name of **Power-Law Ratio**.

To bring some historical ordering to the entries in Table 3, distinctive periods where single variables take similar values in adjacent time periods are enclosed in bold rectangles to demarcate similarities. Columns eight and nine of the table are added to give the values of *q* and the power-law asymptotic slopes where *q*>1. These rectangles reflect relative constancy within limited time periods that define time periods with considerable similarity in these variables:

1. 900
2. 1000-1100
3. 1150-1250
4. 1300-1500
5. 1550-1700
6. 1750-1900
7. 1925-1970

The data in the periods represented by these dates will provide some ordered material for consideration of historical interpretability of the parameters in the *q*-entry model.

Evaluation of Hypothesis 2: Power laws are not appropriate for city-size distributions, beyond the tails, and while the log-normal distribution is appropriate only if growth rates were uniform for cities of different sizes, its cumulative distribution is wrongly shaped.

(a) ***Power law fit***

Hypothesis 1 was concerned with the body of the city size distribution, up to but not including the largest city. Hypothesis 2, concerning the inappropriateness of power laws beyond the tails of city-size distributions, is supported in virtually every study of city sizes. Power-laws do not extend over the bodies of city size distributions, and cannot compete in that respect with the *q*-exponential. Power laws are also unstable because of the ambiguity in choosing the part of the tail that is fitted. The smaller the tail that is fitted, the fewer cities there are in each bin and the greater the unreliability. In the choice of tail is a hidden third parameter of the power "law." In our model there is no ambiguity about the portion of the data that "fit" the model: all the data are used, rather than used selectively.

Figure 4 shows power-law fits for many our historical periods. Since these are among the 75 world's largest cities in each period, these fits are not unlike what is commonly seen in the city-size scaling literature. The fits, averaging about R^2=.93, are not as good as those of *q*-entropy. Bins with single cities at the tails of these distributions were not included in fitting either of the distributions.

(b) ***Body versus Tails***

The other dimension of power-law fit that few studies have examined is how well power laws found in the tails of city size distributions fit with the *q*-exponential bodies. This is a question studied by Borges (2004, 2005) within the *q*-exponential framework, although not for city sizes. Figure 5 examines the tails of our data distributions by restoring the actual population size data for the Chinese city data fitted in Figure 2(a) where we removed each primate city in the tails in fitting the distributions (see FN 5). The raw data points are plotted with straight lines between them in Figure 5, with each of their *q*-curve fits superimposed.

The most common pattern evident in Figure 5 for the raw data is that of capital or leading cities that appear not to follow the *q*-exponential or the power-law. Actually, however, the abrupt transitions horizontal lines in the tails of these distributions are consistent with Zipfian distributions where the largest city is twice the size of the next. This seeming discontinuity was introduced by choice of bin sizes. It takes three of our bins (successive multiples by $^3\sqrt{2}$) for doubling in size, and we needed small bin multipliers to discriminate bins in the body of the distribution. The horizontal lines in the tails, then, actually fit the power law and fit the *q*-entropy asymptotes to power-law tails but in fitting both functions and making R^2 goodness-of-fit calculations we excluded the primate city.

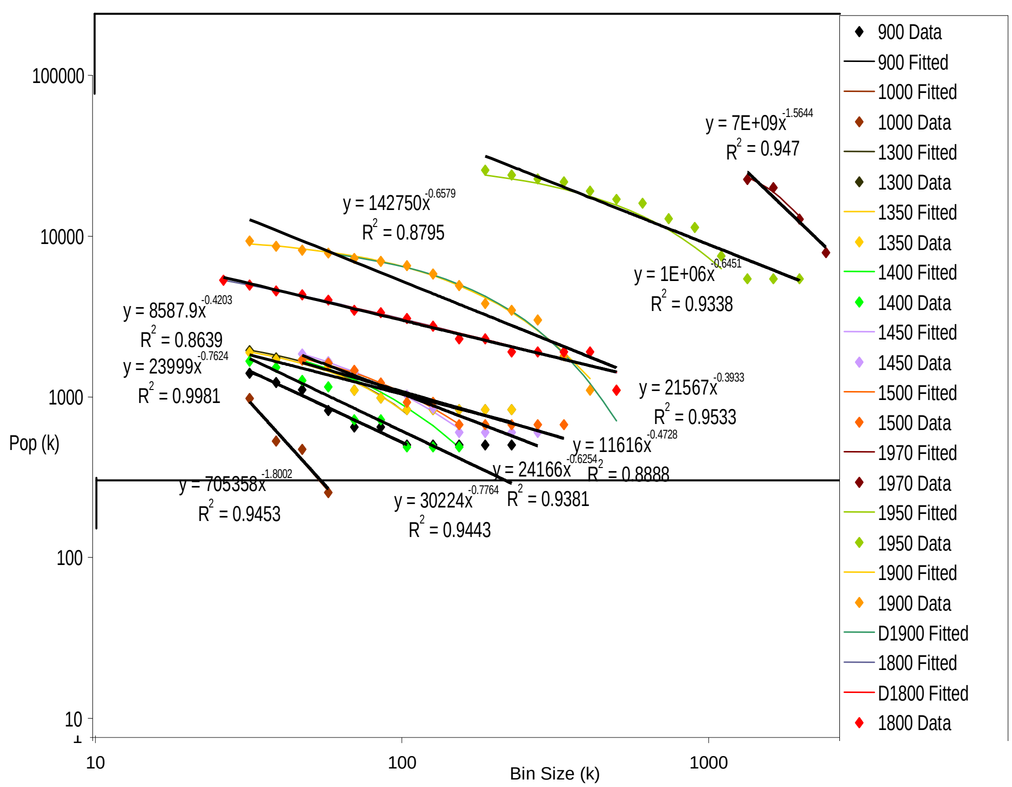


Figure 4: Chinese Cities, some fitted power laws with R^2, averaging=.93

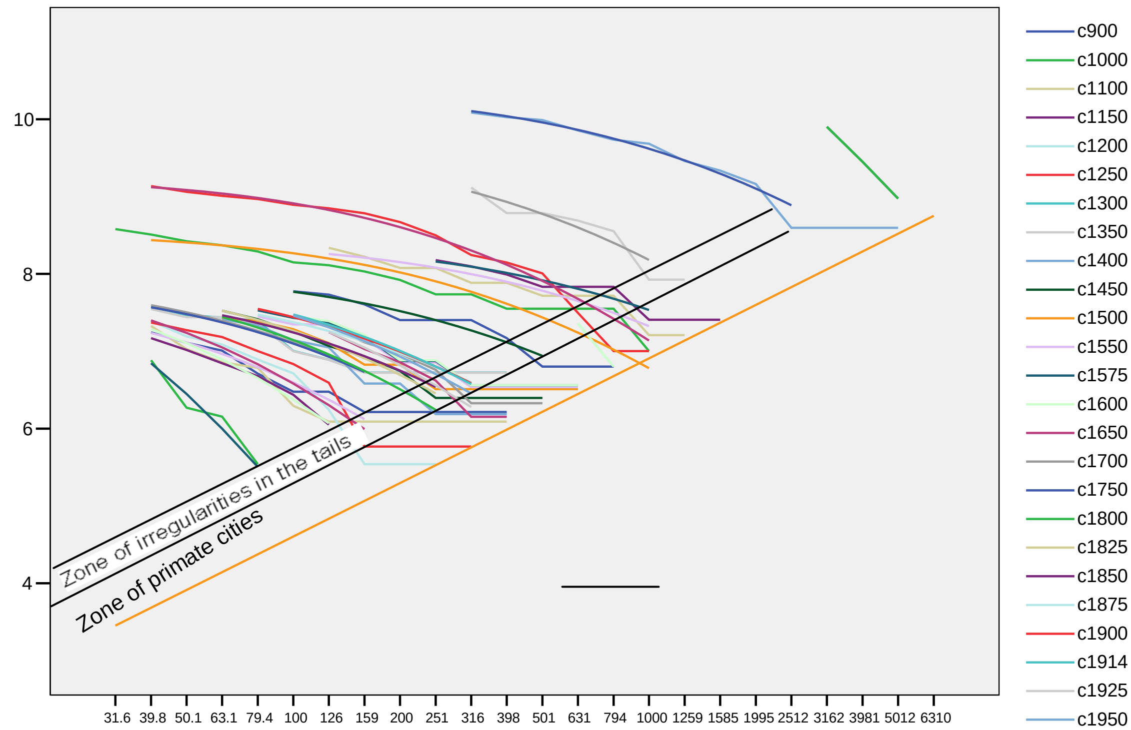


Figure 5: Chinese Cities, fitted *q*-lines and actual population size data[10]

(c) ***Lognormal fit***

An alternate model for city sizes suggested by some researchers is the log-normal distribution, which is fit with the equation $Y \equiv a * e^{-0.5(\ln(x/x0)/b))^2}$. The underlying assumption is one of many independent multiplicative errors. This assumption might be appropriate, for example, if growth rates are uniform for cities of different sizes. Somewhat like assuming a Y0 total urban population. the left side of the log-normal tapers off, which would represent "urban population" tapering off at lower community sizes. Although Excel Solver and Spss nonlinear regression converge to give fits to this model, each of the three parameter time-series correlated positively with the year (R=.75, .57, and .46) and failed to show historical periodicity. Parameters *a* and *x0* correlate with κ in the *q*-exponential (both increase through time), and *b* correlates negatively but moderately with Y0. A moderate temporal prediction is that *b* correlates negatively with detrended population 50 years later. A provisional hypothesis is that the log-normal fits better in the early period up to 1300. This model also fits better than the *q*-exponential for the years 1100 through 1300 where the data series are compact in number of bins covered by the data and our CCFD curves are relatively straight in the log-log graph of Figure 2(a). This did not hold for 900-1000, 1350, or later, and many of the later series were nonconvergent in Spss nonlinear regression.

We also used Excel Solver to fit the cumulative lognormal probability distribution $Y \equiv \frac{a}{2} erfc\left(\frac{-\ln(x/b)}{\sqrt{2}c}\right)$. The parameter values for the fits do not correlate with those of the first equation nor with the year, and b was the only parameter to have historical periodicity (marginally significant at p=.089, computed by the runs test), as is somewhat evident in Figure 6, which plots the values of b through time. In this test we normalized the unit

[10] Figure 3 shows that bodies and tails of historical city-size distributions also might be considered separately.

probability by dividing by P, the total population. Next, we divided by Y0, estimated urban population, to normalize the pdf. The b value for these fits correlated near-perfectly with the b in the previous model (R=.993; R=.84, .92 for a and c) and Figure 6, but with historical periodicity significant at p=.039.

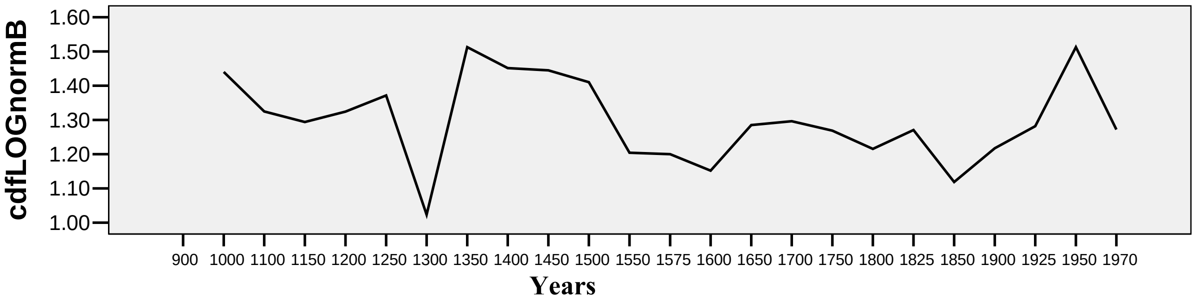


Figure 6: Parameter *b* fitted from the cumulative log-normal probability distribution

Overall, our conclusion was the cumulative lognormal distribution might fit parts of the city size distributions in certain years but the lognormal assumption is not true in general. It often has the wrong shape to fit the body of the distributions that we observe historically. But using Y0 from the *q*-entropy model to normalize the cumulative probability distribution did produce statistical runs that might prove useful to historical analysis in ways that would need further investigation.

Evaluation of Hypothesis 3

Hypothesis 3 anticipated that *q*, κ, and Y0 are historically interpretable. At the end of evaluating hypothesis 1, we saw that all seven of the variables in Table 3 contribute to identifying distinctive periods where single variables take similar values in adjacent time periods. At the end of that discussion we identified six periods in terms of the variables in Table 3. Looking at these in terms of the patterns in Figure 2b, we divided period 5 into two to arrive at the following seven periods:

1. 900
2. 1000-1100 <u>*q*=Zipfian period</u> climax of Song innovation
3. 1150-1250 decline to *q*=1 and *q*=0 (overly decentralized)
4. 1300-1500 rise to *q*= Zipfian (400 years from high at 2)
5. 1550-1700 <u>*q*=Zipfian period</u> climax of Qing (200 years from high at 4)

6a 1750-1800 rise to *q*=3 (overly centralized)
6b 1850-1900 decline to *q*= Zipfian
7. 1925-1970 <u>*q*=Zipfian period</u> Maoist climax (270 years from high at 4)

Part of the interpretability of κ is that when it is detrended (or detrended for estimated urban population Y0) it is inversely correlated to *q* up to 1800, with historical periodicity in its oscillations, corresponding to periods 1-5a about. After 1800, that correlation no longer holds and κ, Y0, and total population rise precipitously.

The interpretability of Y0 is that it is an estimate of the total urban population. As we have seen from the evidence of commensurability in Table 3, it has construct validity as an estimate, including a reasonable estimate of the percent urban population in China from 1100 forward.

Evaluation of Hypothesis 4: There is an interactive dynamic between population cycles and *q*-oscillations. Inspection of Table 4 shows that low-*q* (<1) occurs is in the down cycle (1250, 1650, 1875-although not shown in Figure 2b), preceded by a peak total Chinese population. This peak of combined rural and urban population, in Turchin's (2005) secular population cycle, predicts resource scarcity, conflict, and subsequent fall in population, which we assume to affect depopulation in leading cities and collapse in slope of the power-law asymptote. Stated in terms of time lag, this translates as total population peak → fall in q (<1) ~ total population down → sociopolitical conflict down → start of a total population rise. This will be visualized in a cycle-like graph below as we examine how q is implicated in secular population cycles.

(a) Secular population cycles

A simple dynamical pattern is evident in Table 3 for the relation between total population cycles, with rises shaded in yellow in column one, and subsequent drops in power-law slopes toward the more linear q (<1) pattern, marked in yellow in column nine. The power law then reemerges (bolded in column eight) before the next population rise:

High total population q~1.5 1000-1100
Population decline following 1200
Conflict and q<1 1250
High total population q~1.5 1550-1600
Population decline following 1600
Conflict and q~1 1650
High total population q~1.8* 1800-1850 (*exceptional)
Population decline following 1850
Conflict and q<1 (excluded from charts) 1875 (1875 was deleted from q-fitting because the fall in city sizes in China left too few in Chandler's top 75 world cities for the analysis.)
High total population q~1.5 1925-1970
Population growth slowdown following 1995; decline ?
Conflict and q<1 ?

So it would seem that in the decline phases of the secular cycles q collapses below 1 following one or two generations (25-50 years). The calendar time between the first and second demographic cycles (rise to rise) is circa 350 years, between the second and third circa 250 years, and between the third and fourth ca. 150 years. These are what Turchin (2003, 2005, 2006) calls secular cycles.

(b) Evaluating* q*~1.5 and q<1

Until this point we remained agnostic about the "meaning" of q values such as 1.5 and their relation to the Zipfian tail as a configuration that is often taken as an indicator of an energized or productive urban hierarchy. The frequent connection between periods of q~1.5 and 'late' high points in population growth associated with heightened competition and growing scarcity (also higher wages and fertility) allows us to make an evaluative judgment. This correlation is maximal with the last date in our "High total population" periods above; the preceding periods most commonly have q~1.35 or lower; $1 < q < 1.5$ is not uncommon in early and middle periods of growth. Following the 'late' periods of growth are the population declines (and sociopolitical conflicts, if we follow Turchin's findings) that have q<1. We build on the first finding in the sections that follow by labeling q in the interval 1.34-1.5 a potentially "optimal" value associated with urban growth and presumably a sign of economic growth, and we include the Zipfian in that potential optimum. We build on the second by labeling values where $q \leq 1$ "stagnant" values associated with urban decline. We label q>1.7 "rigid" because the power tails are thin, most cities are near the average rather than spreading toward the tails, and the primate city is expected to be too large to be Zipfian.

(c) Conflict and Markets

Conflicts and conquests are connected to world-system dynamics and turning points (Modelski and Thompson 1996), but relative to the internal sociopolitical instabilities that Turchin endogenizes into the oscillatory patterns of secular cycles, external conflicts and external trade have the status of external variables. Hence this one part of our discussion will present some relevant description in terms of narrative (derived from Gascoigne 2003 and Temple 1986) that highlights the importance of these variables relative to q. The Sung q is high (1.90) in 900 just before the period of Sung national markets and monetary currencies develops, but q is nearly Zipfian in 1000 (1.37), rising with population in 1100 (1.72). The Jin conquer the Sung capital of Kaifeng, in the north, in 1127, with a 2/3rd drop in Y0 given loss of the capital, a 15% drop in total population, and a drop in q from 1.72 to 1.40, but Sung innovation, economic florescence, and population growth continues and reaches its maximum in 1200 although monetary inflation ensues and q deflates to .54, the "chaotic" regime of Table 6. By 1250 the Yuan Mongols have mobilized the navies of North China and Korea and q falls to the all-time low of .02 as people flee the large cities. Desperate, the So. Sung allies with the Mongols against the Jin, with Yuan conquest ensuing in 1279. In 1300 and 1350 q has risen to .85 ("random"). Nanjing becomes a center for textile production for an international market as the Mongols control the Silk Routes. Ming resistance retakes China in 1368. By 1400 q reaches 1.24 and the Zipfian q~1.5 is reached in1450. Ming naval power has been enormous but the navy and the huge maritime expeditions are retired in 1434, after which q recedes in 1500 (1.34) and 1550 (1.04) but then recovers in 1575 (1.35) and 1600 (1.48) to a nearly Zipfian level as an insular Chinese facilitates population growth even as the use of currency is abandoned. There follows the economic crash before 1650 induced by overpopulation. At this point major European banks (Hamburg, Stockholm) have adopted the currency and banking systems enhancing international trade that the Chinese have abandoned, and New York banks followed suit in 1690, then Paris (1720), while China q is still stagnant at 1.0 in 1700. The Qing takeover of a stagnating Ming empire by the conquest of Beijing in 1644, however, begins to revitalize and expand the empire.

Hypothesis 5: In the interactive dynamic between population cycles and q-oscillations, κ should play a role along with q in the population recovery phase because it is involved in the *extent* of the reemergent power law.

Table 5 shows a linear regression in which a *low* κ or crossover at a lower city size to a power law tail, and a *low* q (<1.5), or steeper tail, predicts detrended total population growth fifty years later up to 1800, as exemplified by the years 1000, 1400, 1575, and 1700 (and even in 1900). The converse, that a *high* κ or later crossover to a power law tail, and a *high* q (>1.5), or shallower tail, predicts detrended total population decline fifty years later, does not predict so well. The hypothesis is supported for periods up to 1800, but not after. The product of κ and q also correlates strongly (R^2=.63) with total population decline fifty years later, up to 1800, The time period up to 1800 is also one in which q varies inversely with detrended κ (R^2=.52). Other factors seem to overtake the joint effect of κ and q on population growth after 1800, perhaps having to do with the reversal of bullion flow between China and the West due to the opium trade and subsequent changes both in terms of internal sociopolitical violence and in relation to the international economy. Questions of dynamics, however, will require further study in relation to secular cycles and the coding, at the very least, of a sociopolitical instability variable over the millennial time frame.

Table 5: Time-lagged regression of detrended population change

	Unstandardized Coefficients[(a)]		Standardized Coefficients	t	Sig.
Adj. R^2=.549 R^2=.575[(b)]	**B[(c)]**	**Std. Error**	**Beta**		
(Constant)	**2.660**	**.127**		**20.945**	**.000**
κ_Excel0	**-.002**	**.001**	**-.742**	**-3.727**	**.003**
q_Excel0	**-.179**	**.056**	**-.634**	**-3.182**	**.009**

a Dependent Variable: LCPDetrend50YrAfter.
b R^2=.575 with the poor population estimates removed for 900 and 1000, with reduced sample.
c B coefficients are the constants in the regression equation Y=2.66 -.002κ -.179 q.

The quantitative results on time lags are consistent with the idea that a *low q* (<1) period is of relative short duration (50 years or less), predicted by a maximum point of (total) population overreach that in producing resource scarcity and sociopolitical conflict produces population decline (and the low q) but predicting in turn a short turn recovery. Mathematically, however, if we turn all the signs around, *high q* (»1) and *high* κ predict population rise, which can and does occur quickly (in a 25 year lag). With a longer lag population rise predicts scarcity, and a repeat of the cycle.

Except for our discussion on conflict and markets we have tried to avoid a narrative validation of the scaling results against events in Chinese history because they are too susceptible to spurious inferences. Instead, we have preferred to validate the connections to Chinese secular cycles through the lens of the q-exponential mathematical model itself. Preliminary findings on some of the time lagged correlations relevant to analysis of historical dynamics as sketched in Figure 7. The numbers on the arrows are simply raw correlations rather than structural equation modeling.

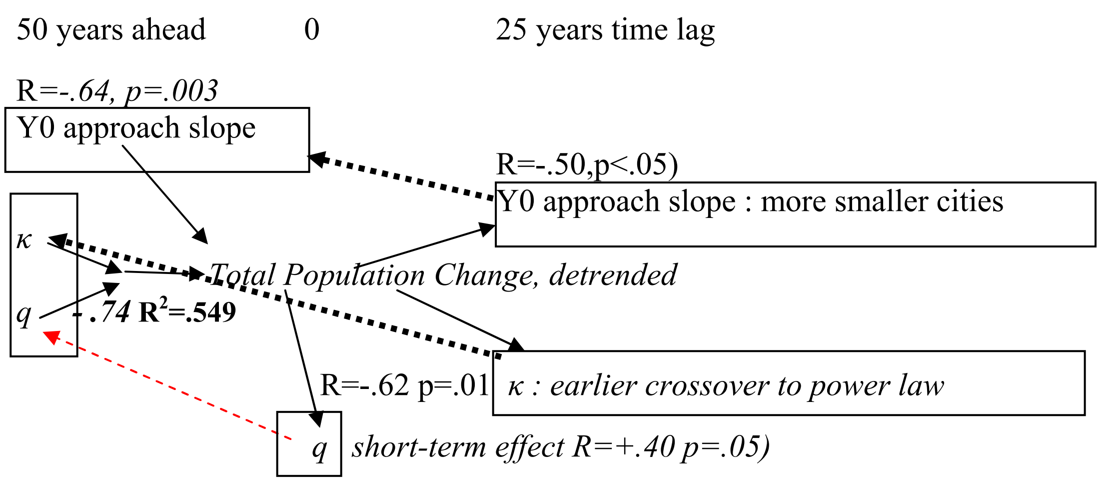


Figure 7: Sketch of some possible lagged correlations relevant to dynamics
(dashed line for negative feedback, dotted link for positive feedbacks)

The sketch in Figure 7 is very far from a complete analysis. It suggests a negative feedback loop for q where total population change (detrended) affects q in a short time frame positively (steeper power-law slope with an earlier crossover), but q and κ combine to influence total population change in a longer time frame of circa 50 years. Other correlations suggest a positive feedback loop, first, for *κ,* and second, for the redistribution of population from large to small cities, and the reverse. This needs to be studied with the endogenous dynamics of the secular population cycle. The complexity of the combined *q* and *κ* feedback also needs to be examined. Those investigations, however, are beyond the scope of this study, which is aimed at discovery and cross-validation, not the dynamical modeling itself.

LINKS TO COMPLEXITY THEORY

Entropy models, q-entropy, and complexity

Entropy models have the advantage of specifying least-effort configurations, in some sense maximally likely given the absence of nonrandom interactions. Knappett. Evans, and Rivers (2006), for example, portray a network of Bronze-Age sites in the Aegean, "with its complicated constraints and interactions, as explicable in terms of an 'energy landscape' … through which the system moves….[11] A system with low energy is close to some optimal solution in which all the different constraints and interactions are balanced." Clarke (1977:20), for example, explaining interest in the use of statistical physics models for archaeology, states "the most probable state of any system at a given time is the one which satisfies the known constraints and which maximizes its entropy."

The simple entropy model is useful as a null hypothesis, assuming random interactions given known constraints. Complex systems, however, are those with nonrandom interactions given constraints. *Q*-entropy models the typical forms that nonrandom interactions take, such as proportionality effects (the more *x* the more *x*, like rich get richer, or proneness to accident) as one direction of deviation from randomness, and in the opposite direction, linearity, which dampens even the continuity effects found in the exponential (like the true binomial, in which the probability of x is successively lower each time it occurs). They are useful because that provide graded departures from simply entropy that may amplify nonlinearity on one side or linearity on the other.

Continuum Conception of Complexity. A manta of complexity theory is that simple principles underlie apparent complexity. In the *q* metric, *q*=1 is a null hypothesis that would imply that we can treat populations as if they were gas molecules, while departures lower than *q*=1 would imply the opposite. A value of *q*>1 asserts mechanisms of attraction, even if they are relatively mild, and approach an exponential entropy associated with complete randomness, while *q*<1 asserts mechanisms that create an asymptote toward linearity in the CCFD. A continuum for describing complex systems is often conceived in terms of balance at an intermediate position between too much rigidity to adapt and too little organizational structure to survive

To reiterate the explanation of Hypotheses 4 (part b) in these terms, we have described an potential optimum for *q* applied to city sizes somewhere the range of 1.5±.15 as implying *mechanisms of attraction that as an organizing principle would generate the commonly observed near-Zipfian tails of urban distributions*. This is the mean for the Chinese historical sequence. Further departure from the Zipfian, where *q*»1.5 entails *an attraction*

[11] Their following statement, that "This is based on a general principle that systems want to minimize their energy (hence conceiving of the system as having agency or behavior of some kind)" is unnecessary. It represents a failure to conceptualize how the agency and behavior of human and other agents connects to sites, populations, transports and the like. It is not the system that "wants" but agents' behavior that may randomly connect to local optima and the exploratory behaviors of agents that often lead them to global optima.

principle that is too rigidly or narrowly concentrated in thin tails. A $q \geq 1.7$ might indicate an overly rigid regime or crisis in which the elite cites are unusually large relative to the average. Further examples are the outliers of $q \approx 3$ for 1800 and 1825 when Britain had a stranglehold on Chinese bullion prior to the Opium wars. At the other pole of this continuum is the disorganization implied by a $q<1$ we might posit *mild attraction to smaller cities* or a size curve that asymptotes from a random-like exponential to a linear distribution. The Chinese city data for years 1200-1250 (and possibly 1875 as well) are examples.

Summary of the patterns of historical relations between secular cycles and q

Part of the historical interpretability we have found is a vocabulary for what is "good" and what is "bad" in relation to population shifts. After study of the China data we offer an interpretive syntheses that is summarized in Table 6 that relates changes in q to oscillations in the total population, or to exceptional conditions such as diversion of bullion flow in the economy through the British opium trade (1800-1825, prior to the Opium Wars) and the deurbanization movement underway in the Cultural Revolution.

Table 6 shows the relation between classifications of the ranges of q and secular population cycles of change or exceptions due to exogenous variables. The dotted lines summarize the regularities in the historical trajectories with large dots for the most common sequences and the small dots for exceptions. Mature total population maxima just before a crash ('late' rise) are associated with q=1.5, which entails Zipfian proportional effects and a possible optimum for creative urban interaction balancing heterogeneity in the body of the distribution with power law nonlinearity in the tails. This is not, however, a stable equilibrium for urban hierarchies. The historical tendency is either to modest rises in q toward a thinner power-law tail increasingly differentiated from the average city ($q \approx 1.7$),[12] or a modest fall toward a thicker tail and greater heterogeneity in the body ($q \approx 1$).[13] The next phase in the secular cycle (following from the pressure of scarcity on resources), civil strife and population decline, is accompanied by fall in q to well below 1. Population recovery in the early rise period in five cases of five is accompanied by approximations to $q \approx 1$. The next step in the population cycle of the first four columns continues from 'early' to 'late' rise, with exceptions in column five and six.

The edge of chaos metaphor of complexity

We mobilize the metaphor of complexity as a "first approximation" to an explanation. It is a truism to say that complexity, life, history, and complex systems generally stand somewhere between rigidity at one pole, exemplified by $q \geq 1.7$, and on the other, randomness ($q \approx 1$), or the heightened unpredictability of chaos ($q<1$).[14] But we do not see support for *equilibrium* on the "edge of chaos" in the data of Figure 2 or Table 6. Our historical periods do tend to cluster, somewhat like "edging on chaos," near $q \approx 1.2$, but they do so in terms of oscillations, not far from equilibrium, but not a stable equilibrium. They also stray into decentralized chaos, here in the metaphoric sense, in which the smaller cities are more attractive than the hubs, or regimes are affected by massive external drains on the economy that seem to put q into abnormally rigid states, or deurbanization attempted as political policy.

[12] This advantage may become so intense if it rises to q=1.7 as to destabilize a complex society. But it is also associated with the sociopolitical conflicts observed by Turchin (2005) in secular population cycles.

[13] The rise occurs in the earlier period with lower asymptotic Y0 slope. In the two instances of fall, on recovers to q=1.5 in the next period and the other is an exceptional shift to radical deurbanization.

[14] Technically, the mathematics assigned to chaos is a deterministic departure from randomness in which a dynamic trajectory never settles down into equilibrium, and small differences in initial conditions lead to divergent trajectories. The link between empirical history and "edge of chaos" is typically done by simulation.

Table 6: Total Chinese population oscillations and *q*

***q* ranges**	**Endogenous secular population cycle**				**Exceptions**	
	'Early' pop. rise	**'Late' pop. rise**	**Population Maximum**	**Crash**	**Economy Captured**	**Exception** ***deurbanized***
***q*~3 'abnormal'**					*1800 2.77* *1825 2.99*	
***q*~1.7 'rigid'**			*1100 1.72* *1850 1.85*			
***q*~1.5 Zipfian**		*1000 1.37* *1450 1.50* *1500 1.34* *1925 1.39*	*1575 1.35* *1600 1.48*	*1150 1.4*		*1970 1.49*
***q*~1 'random'**	*1300 0.85* *1350 0.85* *1400 1.24* *1700 1.00* *1750 1.29* *1900 1.14*		*1550 1.04* *1950 1.06*			
***q*~.5 - .8 'chaotic'**			*1200 0.54*	*1650 0.8* *1875 <1?*		
***q*~0 'flee the cities'**				*1250 0.02*		

Alternative Possibilities

The notion of the Zipfian *q*~1.5 as an optimum for population growth and economic innovation is supported by periods like 1000 CE for the height of the Sung urban renaissance and invention of national markets, and the Ming revival of 1450-1500 following the move of the capital to Beijing. Zipfian *q~1.5* also accompanied the periods of unsuccessful economic reforms of the late Ming period (1550-1600) that preceded population decline.

We need also to consider, then, from other studies, the periods with *q* closer to 1, such as 1300-1350 in the Mongol Yuan revival of textile production, periods like the early Ming (1400) and possibly others (1550, 1700-1750, 1900, 1950) for how their *q* values might relate to economic innovation and generativity. These cases lie, as in Table 6, either at the start of a population rise or a subphase in a mature but stable population maximum.

INITIAL SUMMARY

Having defined the *q*-entropy model and explored its fit to Chandler's Chinese city data, we find ample evidence for the reliability and robustness of estimates of model parameters, and for goodness-of-fit. Spss nonlinear regression and Excel Solver results are convergent and replicable. Given the evidence of reliable scaling, we find support for our five hypotheses:

1. The model is appropriate for the entire body of city size distributions historically, excluding the largest city or cities if they have extraordinary size due to status as regional capital or primate city of a size beyond that expected from a Zipfian.
2. Power-laws have little utility for historical comparisons.
3. The parameters *q*, κ, and Y0 are historically and meaningfully interpretable.

4-5. There are strong signs of endogenous dynamical processes that drive oscillations in the *q*-entropy parameters that are connected to Turchin's secular population cycles.

Once we put all the parameter estimates together in an ordered series—total population, estimated urban population, estimated cumulative population in the smallest of our city-size data bins, and the scale parameter κ that is measured in urban population units—and considered the ratios between them and with *q*, we could identify major historical periods and changes in Chinese history that make sense for city size and population distributions.

We did not attempt here to give a qualitative assessment of the corollary "events" and "periods of rise and fall" in Chinese history, but this could be done using Modelski and Thompson (1996) as a guide. Instead, we noted that our data suggest dynamical interaction between secular cycles of population rise and fall as studied by Turchin (2003, 2005, and 2006) and time-lagged rise and fall in characteristic urban distributions. These relationships take us into a domain of study that is beyond our focus here. Fuller data compilations on China by Turchin and Nefedov (ms. 2006) for the last millennium will be needed.
Our initial look at dynamics suggested that rises and falls in total Chinese population (detrended) both lead and follow changes in *q*-entropic model parameters *q* and κ and other variables derived from relations among model parameters and empirical estimates of urban population characteristics concerning the body and smaller-sized cities in the urban size distribution. What we have discovered are (1) the reality of historically meaningful changes in city size hierarchies that relate to the primary dimensions of change in Chinese history as measured by *q* exponential curves, and (2) the meaningfulness, interpretability, and interrelatedness of the *q*-entropy model parameters.

In doing so we have begun to glimpse how the *q* parameter itself and the theoretical concepts of the *q*-exponential and *q*-entropy relate to salient concepts in complexity theory concerning alternations between "too deterministic" an interactive system falling back to a productive but difficult to sustain center of the *q*-exponent with a more Zipfian gradient indicating mechanisms of attraction and growth after a crossover region in the distribution, possibly giving way at the other extreme to more chaotic "too random" interactions to be sustainable without further structural change. In historical fluctuations, however, not only are economic and market processes not at equilibrium but they couple with internal and external conflict, as shown in Turchin's studies of historical dynamics.

Up to here we have primarily been studying *structure* and structural/demographic measures that are extraordinarily useful to study change, especially for the construction of a suitable database for the study of change. This approach will then equip us in further studies to analyze hypotheses quantitatively about *dynamics*.

WHERE DO WE GO FROM HERE?

Phenomenology

To preface a study of how *q*-entropy is implicated in historical dynamics, although we do not undertake this here, we need to start with what we know phenomenologically.

1. City systems are bounded and constrained within a total population, rural and urban. In China, this is particularly important because of its huge rural agrarian population. Turchin's model of agrarian demography in relation to sociopolitical structure and sociopolitical instability is particularly important. He studied the two dynastic cycles previous to our period from 900 CE: those of the Han and the Tang, shown In Figure 8 (Turchin 2005). Here, population rise and fall leads sociopolitical instability with a lag time of 30 years,

while sociopolitical instability *negatively* leads population with the same time lag. Times between minima and between maxima are between 200-300 years.

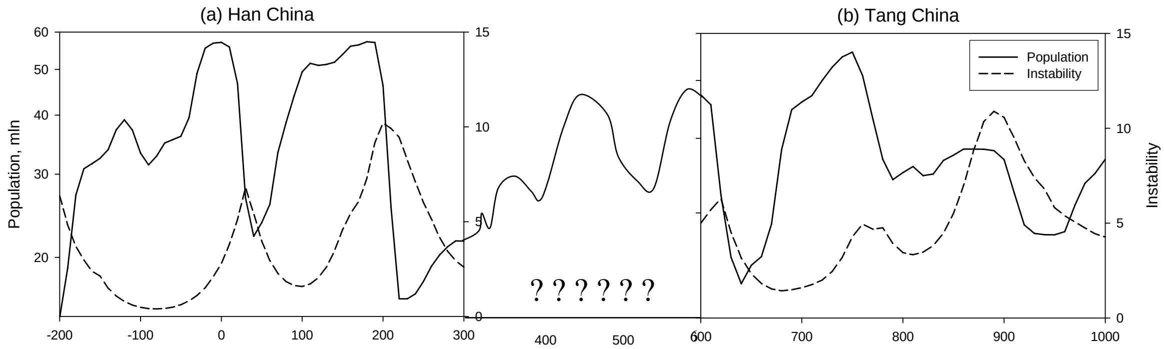


Figure 8: Turchin secular cycles graphs for China up to 1100

Note, Figure 8: (a) and (b) are from Turchin (2005), with population numbers between the Han and Tang Dynasties filled in. Sociopolitical instability in the gap between Turchin's Han and Tang graphs has not been measured.

Turchin's focus has been on secular cycles in agrarian empires, and not on modern market economies and their polities. What happens with a market economy as it develops in China in the startup of our study period and the modern era of national markets circa 1000?

2. City systems are self-bounding in that they begin at a certain settlement size, a size that supports the specialized functions of intercity trade. It is the combination of the intercity trade network and the rural/urban exchanges that supports the intercity trade network. A rural population can subsist on its own, without the urban component, although at a much lower population level. Its units have some autonomy in that a rural settlement can subsist on its own. In the intercity trade network, no city can survive on its own without the network. Modelski (2003:3) explicitly defines a *world city* in this sense as "a community with a significant degree of division of labor that makes it part of a network of cities." And secondly, "urban sites that exhibit complexity of organization," "sites that viewed as an ensemble, constitute the 'center' of the world system."

3. The *q*-entropic model estimate is that *cities begin at settlement size 10,000*. We arrive at this suggestion because in estimating Y0, the asymptote for the total urban population is only a small fraction of the total population (Table 3). Only a small fraction of China's population has been urban, up to the present. But our estimate of Y0 also allows us to calculate the bin size of cities at which *q*-entropy population estimates asymptote to Y0. That determination is easy to produce: at bin size 12.5 thousand our estimates average 90% of Y0 for each period except 1970, with no particular trend (for 1970 the estimate is 3%).[15] The asymptote of Y0 to the "completed numbers" of city dwellers ends at the estimated bin size for bin 10K, or city size 10,000. This aligns with Modelski's (2003:5) review of the archaeological evidence for first cities as part of city networks (Algaze 2001, 2005) as distinct from towns or villages: "For the ancient era, a world city is one with an estimated population of at least 10,000."

[15] This might signal that something is wrong with the parameters—*q*, Y0, and κ—estimated for that period, but the percent urban in 1952, for example, reported as 12.5% based on population of 573 million, includes towns with more than 3,000 residents (Heilig 1999: urban_5.htm, urban_8.htm). Thus if only 7% of the population lived in towns between 3,000 and 10,000, then our Y0 estimate and our estimate of 5.5% urban would be accurate for 1950. By 1970 our % urban estimate falls to 2.2%, but at the time of the Cultural Revolution many urban residents were sent to villages. The 17.4% urban 1970 census report could contain 15% in small towns. For the age-cohort transition in China's current population, see Heilig's (2006) http://www.china-profile.com.

Our criteria for inclusion of cities in our study are broader than those of Modelski for world cities, however. He goes on to say: "For the classical era, a world city is one with an estimated population of at least 100,000." The classical era for Modelski is 1000 BCE to 1000 CE. Many of our cities – the largest of the cities in the world at each time period, for which Chandler collected data – are not only under that threshold (down to size 40,000) but belong to the modern era after 1000 CE, for which Modelski's minimal size for a world city is one million.

4. Our phenomenology shifts away from exclusive concern the power law outliers of city distributions to the whole distribution, down to what are small cities (now "towns") of 10,000. If we restricted our study to "world cities" in the modern era there would be one in China in 1100 CE, two in 1100, 1200, and 1300, two in 1800, and only one in 1900 Modelski's (2003:63), that is, only the extreme outliers in our Figure 4. What we have shown is that it is not just the hubs or the "central city" of each regional component of the world system in a given period that is important, but the entire distribution of cities. Our estimate of Y0 establishes that it is reasonable to assume that size of 10,000, throughout the modern era, is the threshold at which communities take part in the urban network, its division of labor and complex intercity exchanges, and its dynamics.

5. Our phenomenology includes the relation of Y0 to total population, where P (rural and urban) embeds Y0 (urban only), the latter composed of settlements of size 10,000 or more, and then rates of growth or decline for each category, which include migration up or down the urban size scale as well as between the rural population and any of those size categories. They also include births (coupled to wages), and deaths (coupled to sociopolitical instability and conflicts). Our city parameters q and κ are trend summaries for the size distribution shape and asymptotic slopes that are net patterns affected by the more microscopic variables and by the summary variables of rates of population growth or decline as they scale across the size categories.[16]

Figure 8 suggests elements of a schematic for a "sufficient statistics" approach that might be used to model the historical urban systems studied in this paper. This entails the concept of using sufficiently high-level macro or aggregate variables, such as total population and regional conflict in Turchin's secular cycles model, at a level of generality that captures important dynamical linkages such as Malthusian pressure for conflict over resources because of scarcities induced by population growth. In our view, fitting the parameters of the q-entropy model helps to capture some important additional macro variables that take us several steps beyond the endogenous secular cycle variables of Turchin's agrarian empire model to describe how changes in the urban hierarchy have macro cause and consequences. Changes away from Zipfian tails that reflect interactive sources of innovation, for example, are likely to have positive or negative effects on growth. What we have not done here, as attempted in previous papers, is to bring in macro-variables from the dynamics of the intercity trade networks, and the event structure of external conflicts.

[16] Internal note: a thesis project could describe how exactly the micro variables affect the macro variables and extend the analysis to the spatial and network aspects of the q-exponential model.

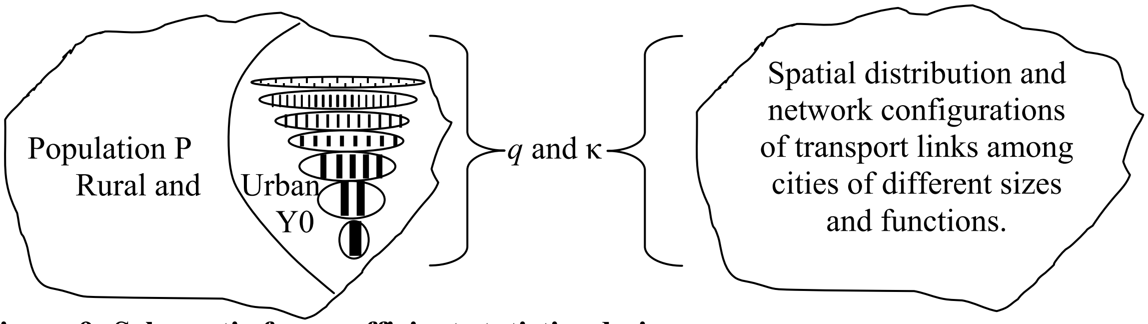


Figure 9: Schematic for a sufficient statistics design

Dynamics: What kinds of variables do we need?

With historical data aggregated and scaled at a high level (P, Y0, q and κ, rates of change, and sociopolitical instability), given a relatively closed system,[17] we actually have a better chance at modeling historical dynamics than by trying to construct extensive microscopic variables. Conventional and micro-history are need to understand detailed processes and inform macro modeling, but as Turchin and others have shown, the importance of macrovariables like total population and resources to support that population define such macrovariables as pressure on resources that are needed to understand historical dynamics within bounded or self-bounding systems. Turchin (2005) is able to show limit cycles that operate between macro pressure on resources and macro sociopolitical instability level as having oscillatory interactions over the secular cycles of 2-300 years. Kohler, et al. (2006) have replicated these cycles for pre-state Southwestern Colorado for the pre-Chacoan, Chacoan, and post-Chacoan, CE 600–1300, for which they have "one of the most accurate and precise demographic datasets for any prehistoric society in the world." Secular oscillation correctly models those periods "when this area is a more or less closed system," but, just as Turchin would have it, not in the "open-systems" period, where it "fits poorly during the time [a 200 year period] when this area is heavily influenced first by the spread of the Chacoan system, and then by its collapse and the local political reorganization that follows." Kohler et al. note that their findings support Turchin's model in terms of being "helpful in isolating periods in which the relationship between violence and population size is not as expected. Relative regional closure is a precondition of the applicability of the model of endogenous oscillation.

Given that caveat, we may be able to model for relatively closed historic periods additional components of Turchin's secular cycles that take into account the effect of markets or economic innovations, and to also model "open system" interactions such as warfare on an interregional scale in terms of their effects on city hierarchies.

[17] As Turchin (2005:18) notes: "The Roman and Chinese empires were largely 'closed' military-political systems by virtue of their size and lack of significant rivals. Even their 'barbarians' can be thought of as an integral part of the system. An excellent case for this interpretation of the Chinese-nomad relationship is made by Barfield (1989)." More generally, the notion of "open" and "closed" as used here is with respect to the variables *endogenized* in the model of internal processes of secular cycles: "Empires … go through an alternation of roughly century-long integrative and disintegrative phases as a result of their inner workings. But this does not mean that external factors are unimportant—… although they do not cause secular cycles, they can influence them" (Turchin 2006:257).

In an "open system" context, our analysis seems to indicate that:
a. External wars in which a Chinese dynasty is invaded and its capital conquered are correlated with a subsequent (or immediate) fall in q below 1.5.
b. External dominance of the economy, such as Britain's diversion of China's bullion flow from opium sale and monopoly, are correlated with a subsequent rise in q above 2.

In the endogenous or "closed system" context, our analysis seems to indicate that:
c. What we take to be higher economic prosperity, innovation, and trade as reflected in the early part of a population growth period may have a positive effect within a generation on pushing the value the population hierarchy closer to one with q=1.5 consistent with Zipfian tails.
d. Peak population resulting from growth, to the extent that it entails pressure on resources, ought to predict with a lag of 50 years a drop to q<1.
e. Simultaneously high q and k reflect mathematically a diminished portion of the tail in the crossover to a power law asymptote, which we take to mean a loss of competitive proportionally effects in the tails. One of our stronger regression findings is that this configuration leads and might predict, by circa 50 years, decreases in urban population.

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